BOOK REVIEW

Integration. By E. J. McShane. Princeton University Press, 1944. 8+392 pp. $6.00.

Modern mathematical analysis deals with many different methods of assigning values to integrals. If A and B are two such methods, perhaps those of Riemann and Lebesgue, we may use the symbols

$$(1) \qquad (A)\int_E f(x)dx, \qquad (B)\int_E f(x)dx$$

to denote the corresponding values of the integrals when E and $f(x)$ are such that the integrals exist, that is, when E and $f(x)$ are such that the methods assign values to the integrals. The following terminology, while not so firmly established in the theory of integration as in the theory of summability of infinite series, serves as a basis for precise statements in the theory of integration. Two methods A and B for evaluation of integrals are *consistent* if the two integrals in (1) are equal whenever both exist. The two methods A and B are *equivalent* if the two integrals in (1) both exist and are equal whenever E and $f(x)$ are such that at least one of the two exists. A method A *includes* a method B if existence of the second integral in (1) implies existence of the first and equality of the two. Thus A and B are equivalent if and only if each includes the other. The esteem attached to the notion of regularity in the theory of summability suggests that a method for evaluation of integrals should be called ***regular*** if it includes the Riemann method.

It could be argued that, in a textbook in applied mathematics in which there is a tacit understanding that only integrals consistent with each other are employed, there is no point in bothering with names of integrals. Whether the integrals be Riemann, Darboux, Lebesgue, Cauchy-Riemann, Cauchy-Lebesgue, Riemann-Stieltjes, and so on, does not matter. In such a book, the statement that a function is integrable means simply that it is integrable by one of the methods in the (undefined) family used. When an integrable function is given, one evaluates or estimates the integral, and proceeds to use the result. In such a book, the statement that a given function is not integrable could not be proved; there is always the possibility that there is some method in the undefined family by means of which the integral can be evaluated. There is no reason why one must stop when he has reached a general Banach integral by means of which every