

x -axis at the time $t=0$. Let $F(x, t)$ denote the resulting temperature distribution at the time $t>0$ according to the equation $4\partial F/\partial t = \partial^2 F/\partial x^2$. For a fixed t , say $t=1/2$, $F(x, t)$ depending linearly on the parameters $\{f_n\}$ is now used as a family of interpolating functions. Indeed, for a given sequence of equidistant ordinates $\{y_n\}$ ($-\infty < n < \infty$), the interpolation problem $F(n, t) = y_n$ ($-\infty < n < \infty$) admits the following explicit solution: (1) $F(x, t) = \sum_{n=-\infty}^{\infty} y_n N(x-n, t)$. Here $N(x, t) = (1/2\pi) \cdot \int_{-\infty}^{\infty} \{\psi(u)/\phi(u)\} \cos ux \, du$ where $\psi(u) = (\sin 2^{-1}u/2^{-1}u)^2 \exp\{-t(2^{-1}u)^2\}$, $\phi(u) = \sum_{k=-\infty}^{\infty} \psi(u+2\pi k)$. For $t=0$, $N(x, 0) = 1 - |x|$ ($-1 \leq x \leq 1$), $N(x, 0) = 0$ ($x > 1$ or $x < -1$), and (1) reduces to the *linear interpolation* of the ordinates $\{y_n\}$. For $t \rightarrow \infty$, $N(x, \infty) = \sin \pi x/\pi x$ and (1) reduces to the *cardinal interpolation series* (2) $F(x, \infty) = \sum_n y_n \{\sin \pi(x-n)/\pi(x-n)\}$. For a fixed finite t , such as $t=1/2$, formula (1) combines the smooth character of (2) ($t = \infty$) with the computational advantages of linear interpolation ($t=0$). The computational advantage of a finite t arises from the exponential damping of $N(x, t)$ as compared with the slow damping of $\sin \pi x/\pi x$. (Received October 2, 1944.)

279. I. J. Schoenberg: *On smoothing and subtabulation of empirical functions by means of heat-flow. II.*

Given a sequence of ordinates to be smoothed and subtabulated; the parameters $\{f_n\}$ of the interpolating function $F(x, t)$ of the previous paper are now determined so as to minimize $S = \sum_{n=-\infty}^{\infty} \{F(n, t) - y_n\}^2 + \epsilon \sum_{n=-\infty}^{\infty} (f_n - y_n)^2$, where ϵ is a positive smoothing parameter. For $\epsilon=0$ the previous interpolation problem results. For $\epsilon = \infty$, $f_n = y_n$ and $F(x, t)$ is the smoothed version by heat-flow of the polygonal line $F(x, 0)$ of vertices (n, y_n) . A compromise between strict interpolation ($\epsilon=0$) and pure smoothing ($\epsilon = \infty$) gives the explicit solution (1) $F(x, t, \epsilon) = \sum_{n=-\infty}^{\infty} y_n N(x-n, t, \epsilon)$, where $N(x, t, \epsilon) = (2\pi)^{-1} \int_{-\infty}^{\infty} \{(\epsilon + \phi(u))/(\epsilon + \phi^2(u))\} \psi(u) \cos ux \, du$. Clearly $N(x, t, 0)$ is identical with $N(x, t)$ of the previous paper. Eight-place tables have just been computed on punched card machines for the function $N(x, t, \epsilon)$ and its derivatives $N'_x(x, t, \epsilon)$, $N''_x(x, t, \epsilon)$ for $t=1/2$, $\epsilon=0, 0.1, 0.2, \dots, 0.9, 1.0$ and the range $-18.5 \leq x \leq 18.5$ (step 0.1) outside of which these functions vanish to 8 places. On increasing the smoothing parameter ϵ , the approximation (1) becomes smoother in the following sense: If $\sum y_n < \infty$ all integrals $\int_{-\infty}^{\infty} \{F^{(k)}(x, t, \epsilon)\}^2 dx$ ($k=0, 1, 2, \dots$) exist and each is a monotone decreasing function of ϵ in the range $0 \leq \epsilon < \infty$. This procedure is being applied to certain empirical functions for which very smooth tables are required. (Received October 2, 1944.)

280. Alexander Weinstein and J. R. Pounder: *On two elementary problems of mechanics and electromagnetic theory.*

It is shown that the problem of the motion of a heavy particle on a rotating earth and the problem of the motion of a point charge in uniform electric and magnetic fields are mathematically equivalent, except for a change of axes; whereas they are usually treated by different methods. (Received September 29, 1944.)

GEOMETRY

281. J. E. Wilkins: *A special class of surfaces in projective differential geometry. II.*

In this paper, which is a sequel to one appearing under the same title (Duke Math. J. vol. 10 (1943) pp. 667-675), a more intensive study is made of surfaces in