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TWO ELEMENT GENERATION OF A SEPARABLE ALGEBRA

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The *minimum rank* of an algebra A over a field F is defined to be the least number $r=r(A)$ of elements $x_1, \cdots, x_r$ such that A is the set of all polynomials in $x_1, \cdots, x_r$ with coefficients in F. In what follows we shall assume that A is an *associative algebra of finite order* over an *infinite* field F.

It is well known that $r(A)=1$ if A is a separable field over F and that $r(A)=2$ if A is a total matric[1] algebra over F. Over fourteen years ago I obtained but did not publish the result that $r(A)=2$ if A is a central division algebra over F. The purpose of this note is to provide a brief proof of the generalization which states that if A is any *separable* algebra over F then $r(A)=1$ or 2 according as A is or is not commutative.

We observe first that a commutative separable[2] algebra Z is a direct sum of separable fields and that there exists a scalar extension K over F such that Z_K has a basis $e_1, \cdots, e_n$ over F for pairwise orthogonal idempotents e_i. If $u_1, \cdots, u_n$ is a basis of Z over F and $x=a_1u_1+\cdots+a_nu_n$ the powers x^i have the form

$$x^i = \sum_{j=1}^{n} b_{ij}u_j \qquad (i = 1, \cdots, n),$$

where the determinant

$$d(a_1, \cdots, a_n) = |b_{ij}|$$

is a polynomial in the parameters $a_1, \cdots, a_n$. If $c_1, \cdots, c_n$ are any

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[1] See page 95 of my *Modern higher algebra*.

[2] The definition of a separable algebra given below reduces to a direct sum of fields in the commutative case. When F is nonmodular the concept of semisimple algebra and separable algebra coincide.