ON ROTATION GROUPS OF PLANE CONTINUOUS CURVES UNDER POINTWISE PERIODIC HOMEOMORPHISMS

DICK WICK HALL

In this paper we make use of the work of G. T. Whyburn[1] on light interior transformations and on orbit decompositions of certain spaces to obtain a theorem by means of which a certain subset of the orbits of points under a periodic transformation $T(M)=M$ may be given a linear ordering. This theorem is then used to obtain an accessibility theorem for plane continuous curves similar to one previously published by L. Whyburn.[2] We take this opportunity to express our indebtedness to G. E. Schweigert for suggesting the proof of Theorem I given below and thus eliminating the longer and less interesting proof previously obtained by the author. For any $x \in M$, the orbit of x under T means $O(x)=\sum_{i=-\infty}^{\infty} T^i(x)$.

THEOREM I. *Let M be a locally connected continuum (that is, a continuous curve) and $T(M)=M$ an arbitrary periodic homeomorphism. Then if a and b are arbitrary points of M lying in different orbits under T and if axb is any simple arc in M joining a and b, then there must exist a simple arc $a'x'b'$ in M lying in the orbit of axb under T such that a' belongs to $O(a)$, b' belongs to $O(b)$ and no two points of $a'x'b'$ lie in the same orbit under T. Furthermore, the point a' may be any arbitrary preassigned point of the orbit of a.*

Proof (Schweigert). Let M' be the hyperspace obtained by decomposing the space M into its orbits under T. Then, since the orbit decomposition is continuous,[3] it follows[4] that there exists a light interior transformation $f(M)=M'$, namely, the transformation given by and associated with the orbit decomposition. Let axb be the given arc in M. Then we may assume without loss of generality that axb has precisely the point a in common with $O(a)$ and precisely the point b in common with $O(b)$. Define $K=f(axb)$. Then K is a locally connected continuum containing $c=f(a)$ and $d=f(b)$. Let cyd be an arc in K joining c to d. Now let a' be an arbitrary point of $O(a)$. Then[5]

Presented to the Society, February 26, 1944; received by the editors April 13, 1944.

[1] See G. T. Whyburn, *Analytic topology*, Amer. Math. Soc. Colloquium Publications, vol. 28, 1942, pp. 182–189 and 239–262.

[2] See L. Whyburn, *Rotation groups about a set of fixed points*, Fund. Math. vol. 28 (1937) pp. 124–130, in particular p. 127.

[3] See G. T. Whyburn, loc. cit. p. 258.

[4] See G. T. Whyburn, loc. cit. p. 130.

[5] See G. T. Whyburn, loc. cit. p. 186.