

A GENERALIZATION OF MOORE'S THEOREM ON SIMPLE TRIODS

GAIL S. YOUNG, JR.

R. L. Moore has proved that the plane does not contain uncountably many mutually exclusive simple triods.¹ The generalization to spaces of higher dimension given below appears to have escaped notice, and to be of some interest.

DEFINITION. If n is a non-negative integer, by a T_n -set we shall mean a continuum which is the sum of an n -cell, g , and an arc, t , such that $g \cdot t$ is a point which is an end point of t and a relatively interior point of g . The point $g \cdot t$ will be called a junction point.

Obviously, a T_1 -set is a simple triod.

THEOREM. Euclidean n -space does not contain uncountably many mutually exclusive T_{n-1} -sets.

PROOF. Suppose that the theorem is false. Then for some positive number ϵ there exists an uncountable collection, G , of mutually exclusive T_{n-1} -sets such that the junction point of each is at distance greater than ϵ from the boundary of its $(n-1)$ -cell. There is a point, P , which is a point of condensation of the set of all junction points of elements of G ; let U be a spherical domain of n -space with center P and radius less than $\epsilon/2$. If the point X of U is a junction point of an element T of G , let $g(X)$ denote the component that contains X of the intersection of U and the $(n-1)$ -cell of T . It is an easy consequence of the Alexander duality theorem that $g(X)$ separates U . Hence the collection, G' , of all sets $g(X)$ is an uncountable collection of cuttings of U , and it is clearly non-separated. By a theorem due to Whyburn,² G' contains an uncountable saturated subcollection, G'' . But if $g(X)$ is an element of G'' , U contains an arc, t , which is in the T_{n-1} -set of G that contains X and which has only X in common with $g(X)$. Since the elements of G are mutually exclusive, no element of G'' separates a point of t from $g(X)$ in U , which is a contradiction.

PURDUE UNIVERSITY

Presented to the Society, April 29, 1944; received by the editors March 25, 1944.

¹ In fact, Moore has proved that the plane does not contain uncountably many mutually exclusive triodic continua. See *Concerning triods in the plane and the junction points of plane continua*, Proc. Nat. Acad. Sci. U.S.A. vol. 14 (1928) pp. 85-88, and *Concerning triodic continua in the plane*, Fund. Math. vol. 13 (1929) pp. 261-263.

² Theorem 2.2 of chap. 3 of his book, *Analytic topology*, Amer. Math. Soc. Colloquium Publications, vol. 28.