CONTRACTIONS IN NON-EUCLIDEAN SPACES

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The existence of an extension of the range of definition of a function $f(x)$ defined on a set S of a metric space M to a metric space M' so as to preserve a contraction of the type

$$\|f(x_1), f(x_2)\|' \leqq \|x_1, x_2\| \tag{1}$$

depends upon M and M'. The author has previously shown [3, 4][1] that for $M = M'$ the extension exists when M is: (1) the n-dimensional Euclidean space; (2) the surface of the n-dimensional Euclidean sphere; (3) the general Hilbert space. In this brief article *the extension is shown to exist when each M and M' is the n-dimensional hyperbolic space.* The method used to prove this result is applied to a metric space which includes both the hemispherical and hyperbolic cases. Hence a unification of results is also obtained.

As shown in the previous papers [3, 4] a necessary and sufficient condition for a contraction to be extensible in M and M' is the property E, which is restated as follows.

PROPERTY E. *Consider in each of the metric spaces M and M' a set of spheres, such that to each sphere $S_i \in M$, having center x_i and radius r_i, there corresponds a sphere $S_i' \in M'$, having center x_i' and radius r_i'. Furthermore suppose that*

$$\begin{gathered} r_i = r_i', \\ \|x_i', x_j'\|' \leqq \|x_i, x_j\| \end{gathered} \tag{2}$$

for all corresponding spheres S_i and S_i', and for all corresponding pairs (S_i, S_j) and (S_i', S_j').

The spaces M and M' are said to have the extensibility property E *if conditions* (2) *and*

$$\prod_i S_i \neq 0 \tag{3}$$

imply that

$$\prod_i S_i' \neq 0. \tag{4}$$

If the above statement holds for $M = M'$, the space M is said to have property E.

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[1] Numbers in brackets refer to references at the end of the paper.