CONJUGATE NETS IN ASYMPTOTIC PARAMETERS

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1. **Introduction.** Conjugate nets on an analytic nonruled surface in ordinary space may be studied either in conjugate parameters or in asymptotic parameters. The purpose of this paper is to make some contributions to the theory of conjugate nets in asymptotic parameters.

M. L. MacQueen has studied the pencil of quadrics each of which has second-order contact with a surface at a point and third-order contact with both curves of a conjugate net at the point. This pencil of quadrics is contained in the bundle of quadrics each of which has contact of at least the third order with both curves of a conjugate net at a point. The equation of this bundle of quadrics is deduced and the bundle is studied in this paper.

W. M. Davis in his Chicago doctoral dissertation, *Contributions to the theory of conjugate nets*, defined and studied several canonical configurations, considering the conjugate net as parametric. This paper presents some of Davis's results translated into asymptotic parameters.

A summary of that portion of the theory of conjugate nets in asymptotic parameters which is used in subsequent sections is given in §2. §3 deals with the bundle of quadrics mentioned above. Certain polar relations with respect to the quadrics of this bundle are presented in §4. In §5 certain loci and envelopes which arise in the study of a pencil of conjugate nets at a point on a surface are studied. §6 presents a study of Davis's canonical configurations in asymptotic parameters.

2. **Analytic basis.** The purpose of this section is to summarize[1] for later use portions of the theory of conjugate nets on an analytic nonruled surface referred to its asymptotic net in ordinary space.

Let the projective homogeneous coordinates $x_1, \cdots, x_4$ of a point P_x on a nonruled surface S in ordinary space be given by the parametric vector equation

$$x = x(u, v). \tag{2.1}$$

A necessary and sufficient condition that the asymptotic net be para-

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[1] E. P. Lane, *A treatise on projective differential geometry*, Chicago, 1942, pp. 89, 105–113, 115–120, 125–129, 150–152, 180–190.