

the integral  $\iint F dx dy$  is equivalent to the condition of irrotationality. Hilbert conjectured that every regular variational problem is solvable. A. Haar showed (Math. Ann. vol. 97, p. 124) that this is indeed the case for the integral  $\iint f(\psi_x, \psi_y)$ . Haar's theorem cannot be applied directly for our case because  $\rho$  is a double valued function of  $\tau$ . Furthermore  $\tau$  has a maximum value  $\tau_s$  at the speed of sound. However one can introduce a hypothetical "semi-compressible" fluid such that (a) for  $\tau < \tau_s - \epsilon$ ,  $p\rho^{-k}$  is (approximately) constant; (b) for  $\tau > \tau_s - \epsilon$  the density is (approximately) constant; (c) the associated variational problem is regular. As a consequence of Haar's theorem *the interior boundary value problem is always solvable for such a semi-compressible fluid. The necessary and sufficient condition for the existence of a subsonic solution of a real compressible fluid is that for the (always existing) "semi-compressible" solution  $\tau < \tau_s$ .* It is believed that the theorem can be extended for the exterior problem too. (Received June 27, 1944.)

### GEOMETRY

#### 235. Herbert Busemann: *Local metric geometry.*

The author studies systematically (not necessarily symmetric) metric spaces in which extremals exist. The local properties of such spaces are investigated first, for instance, segments are constructed which may replace the line elements of differential geometry. Convergence of extremals as point sets, sets of line elements, and curves is analyzed. Then a theory of parallelism between infinite rays in unbounded spaces is developed which is new also for the differentiable case. The theory of locally isometric spaces proves to be analogous to the topological theory of covering spaces with one noteworthy exception: compact and locally isometric spaces are congruent. Finally some of the fundamental theorems of differential geometry on spaces with constant curvature are derived without any differentiability assumptions. (Received June 17, 1944.)

#### 236. Claude Chevalley: *Intersections of algebraic and algebroid varieties.*

The paper contains a local theory of intersections for algebroid and algebraic varieties. The definition of intersection multiplicities is based on the notion of multiplicities of a local ring with respect to a system of parameters as introduced in the author's paper on *The theory of local rings*, Ann. of Math. vol. 44, no. 4. (Received July 1, 1944.)

#### 237. John DeCicco: *Conformal maps with isothermal systems of scale curves.*

Let a surface  $\Sigma$  be mapped conformally upon a plane  $\Pi$ . The scale function  $\sigma = ds/dS$  depends only upon the position of the point. A scale curve is the locus of a point along which the scale  $\sigma$  does not vary. Under any conformal map of  $\Sigma$  upon  $\Pi$  there are  $\infty^1$  scale curves (whereas in the nonconformal case there are  $\infty^2$  scale curves). Any family (isothermal or not) of  $\infty^1$  curves may represent the scale curves of a conformal map of some surface  $\Sigma$  upon the plane  $\Pi$ . The author considers the surfaces  $\Sigma$  of non-constant gaussian curvature which are applicable upon a surface of revolution for which there exists a conformal map of  $\Sigma$  upon  $\Pi$  with isothermal families of scale curves. Of course, any such surface  $\Sigma$  may be so mapped. In that case, the scale curves must be parallel straight lines or concentric circles. It is proved that there