

$k$  and has exponent of  $p$  of  $-1$  if  $p-1$  divides  $k$ . This paper generalizes this result, proving that all the Schur derivates  $\Delta^m S[n, x^k]$  are  $p$ -adically bounded with exponent of  $p$  not less than  $-2m-1-m/(p-1)$  and hence  $p$ -adically convergent. Formulas for  $\lim_{n \rightarrow \infty} \Delta^m S[n, x^k]$  are given in terms of  $\lim_{n \rightarrow \infty} S[n, x^{k-m}]$ . For positive  $k$ ,  $\lim_{n \rightarrow \infty} S[n, x^{2k+1}] = 0$  and  $\lim_{n \rightarrow \infty} S[n, x^{2k}] = (-1)^{k-1} B_k(1-p^{2k-1})$ , where  $B_i$  is the  $i$ th Bernoullian number. The Schur derivates of  $\{S[n, f(x)]\}$ , where  $f(x) = \sum_{i=2}^{\infty} a_i x^i$  and the valuation of  $a_i \rightarrow 0$  as  $i \rightarrow \infty$ , are  $p$ -adically bounded and convergent; moreover  $\lim_{n \rightarrow \infty} S[n, f(x)] = \sum_{i=2}^{\infty} a_i \lim_{n \rightarrow \infty} S[n, x^i]$ . (Received June 29, 1944.)

### 205. Gordon Pall: *Note on factorization in quadratic fields.*

It is proved that if the quadratic integer  $x_0 + x_1 \omega$  is primitive, that is  $x_0$  and  $x_1$  are coprime, then the divisors of  $x_0 + x_1 \omega$  of a given norm are uniquely determined up to a unit factor. Conditions are obtained for the existence of factors of a given norm. It is claimed that the necessity for the introduction of ideals should be based not on the statement that factorization is not unique, but rather that factors do not exist. Thus in the arithmetic of ordinary quaternions, factorization of imprimitive quaternions is not unique, but that of primitive quaternions is both possible and unique; and ideals are in that case unnecessary. (Received July 10, 1944.)

### 206. R. R. Stoll: *Primitive semigroups.*

Let  $F$  denote the class of semigroups  $S$  each of whose elements  $s$  satisfies an equation of the form  $s^n = s^m$  ( $n > m$ ). A semigroup  $S \in F$  is called primitive if for each idempotent  $e \in S$  there exists no idempotent  $f \neq e$  such that  $ef = fe = f$ . Examples of such semigroups are (a) semigroups of  $F$  which contain only one idempotent and (b) semigroups containing a zero and such that each element is nilpotent (nil semigroups). The following structure theorem is proved for primitive semigroups. A primitive semigroup  $S$  contains a unique minimal ideal  $M$  with these properties: it is a completely simple semigroup without zero (Rees, Proc. Cambridge Philos. Soc. vol. 36 (1940) pp. 387-400), and the difference semigroup of  $S$  modulo  $M$  is a nil semigroup. Conversely, a semigroup  $S \in F$  with this structure is primitive. (Received July 10, 1944.)

## ANALYSIS

### 207. R. P. Agnew: *Abel transforms of Tauberian series.*

Let  $\rho_1 = .9680448 \dots$ ; the constant is Euler's constant plus  $\log \log 2$  minus  $2E(-\log 2)$ . The following assertion is true when  $\rho \geq \rho_1$  and false when  $\rho < \rho_1$ . Let  $u_0 + u_1 + \dots$  be a series satisfying the Tauberian condition  $n|u_n| < K$ . Let  $L$  be the set of limit points of the sequence of partial sums of  $\sum u_n$ . Let  $\sigma(t) = \sum t^k u_k$  be the Abel transform of  $\sum u_n$ . Let  $L_A$  denote the set of limit points of  $\sigma(t)$ ;  $z'' \in L_A$  if there is a sequence  $t_n$  such that  $0 < t_n < 1$ ,  $t_n \rightarrow 1$ , and  $\sigma(t_n) \rightarrow z''$ . To each  $z' \in L$  corresponds a  $z'' \in L_A$  such that  $|z' - z''| \leq \rho \limsup n|u_n|$ . (Received July 19, 1944.)

### 208. R. P. Agnew: *A genesis for Cesàro methods.*

The family  $C_r$  of Cesàro methods of summability,  $r \neq 0, -1, -2, \dots$ , is and can be defined as the unique class of methods of summability whose members are simultaneously Nörlund methods and Hurwitz-Silverman-Hausdorff methods. The only methods simultaneously Riesz methods and Hurwitz-Silverman-Hausdorff methods are methods  $\Gamma_r$  closely related to the methods  $C_r$ . (Received June 16, 1944.)