ON UNIFORM CONVERGENCE OF FOURIER SERIES

OTTO SZÁSZ

1. Introduction. In this section we collect some known concepts and simple facts, pertinent to our subject.

Given a sequence of real numbers s_n, $n \geqq 0$, consider for any $\lambda > 1$

$$\limsup_{n\to\infty} \max_{n<m\leqq\lambda n} (s_m - s_n) = u(\lambda) \leqq +\infty;$$

clearly $u(\lambda)$ decreases as $\lambda \downarrow 1$; if

$$\lim_{\lambda\to 1} u(\lambda) \leqq 0, \tag{1.1}$$

then the sequence $\{s_n\}$ is called slowly oscillating from above; similarly slow oscillation from below is defined by

$$\lim_{\lambda\to 1} \liminf_{n\to\infty} \min_{n<m\leqq\lambda n} (s_m - s_n) \geqq 0. \tag{1.2}$$

If both (1.1) and (1.2) hold, that is if

$$\lim_{\lambda\to 1} \limsup_{n\to\infty} \max_{n<m\leqq\lambda n} | s_m - s_n | = 0, \tag{1.3}$$

then the sequence is called simply slowly oscillating. If $s_n = \sum_0^n a_\nu$ is the nth partial sum of a series $\sum_0^\infty a_\nu$, then (1.3) can be written as

$$\lim_{\lambda\to 1} \limsup_{n\to\infty} \max_{n<m\leqq\lambda n} \left| \sum_{n+1}^{m} a_\nu \right| = 0. \tag{1.4}$$

A more restricted class of series is defined by

$$\lim_{\lambda\to 1} \limsup_{n\to\infty} \sum_{n<\nu\leqq\lambda n} | a_\nu | = 0. \tag{1.5}$$

Special cases: If for some $p>0$, $n|a_n| < p$ for all n, then

$$\sum_{n<\nu\leqq\lambda n} | a_\nu | < p \sum_{n}^{\lambda n} \frac{1}{\nu} = O(\log \lambda).$$

Hence (1.5) holds.

If only

$$na_n > -p \qquad \text{for all } n,$$

then

Presented to the Society, September 12, 1943; received by the editors January 19, 1944.