THE DETERMINATION OF SOME PROPERTIES OF A FUNCTION SATISFYING A PARTIAL DIFFERENTIAL EQUATION FROM ITS SERIES DEVELOPMENT

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1. The method of integral operators. The solution of an equation and its associate. The method of integral operators in the theory of linear partial differential equations of the type[1]

$$\begin{aligned} L(U) &\equiv (U_{xx}+U_{yy})/4 + A(x,y)U_x/2 + B(x,y)U_y/2 + C(x,y)U \\ &\equiv U_{z\bar{z}} + 2\,\mathrm{Re}\,[a(z,\bar{z})U_z] + c(z,\bar{z})U = 0, \end{aligned} \tag{1.1}$$

where

$$z = x+iy, \qquad \bar{z} = x - iy, \qquad U_z = [(\partial U/\partial x) - i(\partial U/\partial y)]/2,$$
$$U_{\bar{z}} = [(\partial U/\partial x) + i(\partial U/\partial y)]/2,$$

consists in associating with an arbitrary analytic function $f(\zeta)$ of a complex variable ζ, by means of an operator of the form

$$U(z,\bar{z}) = \boldsymbol{M}(f) \equiv \mathrm{Re}\,[\boldsymbol{P}(f)], \tag{1.2}$$

$$u(z,\bar{z}) = \boldsymbol{P}(f) = \int_{-1}^{1} \mathrm{E}(z,\bar{z},t) f(z(1-t^2)/2)\,dt/(1-t^2)^{1/2}, \tag{1.3}$$

a solution $U(z,\bar{z})$ of the equation (1.1).

$\mathrm{E}=\mathrm{E}(z,\bar{z},t)$, $|t| \leqq 1$, is any analytic function of z and $\bar{z}$ which satisfies the equation

$$G(\mathrm{E}) \equiv (1-t^2)(\mathrm{E}_{\bar{z}t} + a\mathrm{E}_t) - t^{-1}(\mathrm{E}_{\bar{z}} + a\mathrm{E}) + 2ztL(\mathrm{E}) = 0, \tag{1.4}$$

is regular in a sufficiently large domain and has the property that $(\mathrm{E}_{\bar{z}}+A\mathrm{E})/zt$ is continuous at $\bar{z}=0$, $t=0$.

REMARK. An operator (1.2) is determined by choosing a particular function E (the *generating function* of the operator) which satisfies the above requirements.

Let $U(z,\bar{z})$ be a function which satisfies the equation $L(U)=0$ and which is an entire function of two complex variables x and y, that is, a solution of (1.1) whose series development

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[1] Since we consider the functions $u(z,\bar{z})$ for real values of x and y, that is for z and $\bar{z}$ conjugate, it would be, of course, sufficient to write simple $u(z)$. We shall, however, use the first notation in order to stress the fact that $u(z,\bar{z})$ is a (complex) analytic function of two *real* variables x, y, reserving $u(z)$ for analytic functions of *one* complex variable.