APPLIED MATHEMATICS

187. A. N. Lowan and H. E. Salzer: *Formulas for complex interpolation.*

Whenever an analytic function is tabulated for arguments z_ν located at equidistant points along any straight line in the z-plane, the application of the Lagrange-Hermite interpolation formula, which approximates the function by a complex polynomial of degree n passing through any $n+1$ points, leads to the sum from $\nu=-[n/2]$ to $\nu=[(n+1)/2]$ of $f(z)\sim\sum A_\nu^{(n)}(p)f(z_\nu)$ where $A_\nu^{(n)}(P)$ are polynomials in $P=(z-z_0)/h$ and h is the complex tabular interval and where $[x]$ stands for the largest integer in x. Since in general z is not on the straight line containing the point z_ν, P is complex $(=p+iq)$. Expressions for the real and imaginary parts of $A_\nu^{(n)}(P)$ as functions of p and q were obtained for the cases of three, four, five and six point interpolation and arranged in a form especially suited for computational purposes. These expressions are particularly applicable to the cases of functions tabulated along rays through the origin, as for instance: the table of the Bessel functions $J_0(z)$ and $J_1(z)$ computed by the Mathematical Tables Project (Columbia Press, 1943), the forthcoming table of $Y_0(z)$ and $Y_1(z)$, H. T. Davis' table of $1/\Gamma(z)$ and Kennelly's tables of complex circular and hyperbolic functions for Cartesian and polar arguments. (Received April 12, 1944.)

188. H. E. Salzer: *Coefficients for mid-interval numerical integration with central differences.*

Coefficients in the formula for numerical integration from mid-interval to mid-interval using central differences were calculated to the extent where one can employ central differences up to the forty-ninth order. (Previous calculations have gone only as far as coefficients of the seventh central difference.) The coefficients K_{2s} occur in the following formula, which is also known as the first Gaussian summation-formula: $(1/h)\int_{a-h/2}^{a+(n-1/2)h} f(x)dx=[f(a)+f(a+h)+\cdots+f(a+(n-2)h)+f(a+(n-1)h)]+\sum_{s=1}^{m-1}K_{2s}[\delta^{2s-1}f(a+(n-1/2)h)-\delta^{2s-1}f(a-h/2)]+nh^{2m}K_{2m}f^{(2m)}(\xi)$. Due to the extreme rapidity with which the coefficients K_{2s} decrease with increasing s, the calculated table can be used with high accuracy in either integration or summation even though successive differences might not show the slightest tendency to decrease for the given interval. The quantities K_2 to K_{20} are given exactly and K_{22} to K_{50} are given to 18 decimals, accurate to within 0.6 units in the 18th place. These coefficients were checked by two cumulative recursion formulas, by differencing of the ratios K_{2s}/K_{2s+2}, and by a numerical example. (Received May 13, 1944.)

189. H. E. Salzer: *Table of coefficients for differences in terms of the derivatives.*

A table was prepared which lists the exact values of the coefficients $B_{m,s}$ for $m=1, 2, \cdots, 20$ and $s=m, \cdots, 20$ in the formula of Markoff which expresses the mth advancing difference in terms of the derivatives according to the equation $\Delta_h^m f(a)=\sum_{s=m}^{n-1}B_{m,s}h^sD^sf(a)+B_{m,n}h^nD^nf(\eta)$ where h denotes the tabular interval. The quantity $B_{m,s}$ in Milne-Thomson's notation is equal to the $(s-m)$th Bernoulli number of order $-m$, divided by $(s-m)!$ and in Jordan's notation $B_{m,s}$ is equal to $m!/s!$ multiplied by $\mathfrak{S}_s^m$ where $\mathfrak{S}_s^m$ is the Stirling number of the second kind. The coefficients $B_{m,s}$ were calculated by first obtaining the Stirling numbers $\mathfrak{S}_s^m$ (using their well