PROJECTIVE INVARIANTS OF INTERSECTION OF CERTAIN PAIRS OF SURFACES

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1. **Introduction.** In a recent paper [2][1] the author has shown the existence, together with metric and projective characterizations,[2] of a unique projective invariant determined by the neighborhood of the second order of two surfaces S_1, S_2 at an ordinary point O in ordinary space when the tangent planes τ_1, τ_2 of the surfaces S_1, S_2 at the point O are distinct and the line t of intersection of the two tangent planes τ_1, τ_2 does not coincide with any one of the asymptotic tangents of the surfaces S_1, S_2 at the point O. On the other hand, with regard to the coincidences of the line t and the asymptotic tangents of the surfaces S_1, S_2 at the point O two essentially different cases can arise. The object of this note is to derive some projective invariants in these cases. Noticing that no projective invariant can be determined by the neighborhood of the second order of the surfaces S_1, S_2 at the point O, we obtain all projective invariants determined by the neighborhood of the second order of one surface and that of the third order of the other at the point O.

I. Two surfaces with distinct tangent planes and distinct asymptotic tangents at an ordinary point

2. **Derivation of invariants.** Let us first consider two surfaces S_1, S_2 in ordinary space intersecting at an ordinary point O with distinct tangent planes τ_1, τ_2, whose line of intersection t is supposed to be an asymptotic tangent of the surface S_1 at the point O. Let t_1 be the other asymptotic tangent of the surface S_1 at the point O, and t_2 the harmonic conjugate line of t with respect to the asymptotic tangents of the surface S_2 at the point O. If we choose the point O to be the origin, the lines t, t_2, t_1 to be respectively the axes x, y, z of a general non-homogeneous projective coordinate system, then the power series expansions of the surfaces S_1, S_2 in the neighborhood of the point O may be written in the form

$$S_1:\quad y = lxz + px^3 + rx^2z + sxz^2 + qz^3 + \cdots, \tag{1}$$

$$S_2:\quad z = mx^2 + ny^2 + \cdots. \tag{2}$$

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[1] Numbers in square brackets refer to the references at the end of the paper.

[2] An extension of these results to two hypersurfaces has been made by Professor Su. See his paper [6].