A RECURRENCE FORMULA FOR THE SOLUTIONS OF CERTAIN LINEAR PARTIAL DIFFERENTIAL EQUATIONS

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1. Introduction. In a number of recent papers, Bergman[1] has developed the theory of operational methods for transforming analytic functions of a complex variable into solutions of the linear partial differential equation

$$L(U) = U_{z\bar{z}} + a(z, \bar{z})U_z + b(z, \bar{z})U_{\bar{z}} + c(z, \bar{z})U = 0, \tag{1.1}$$

where $z=x+iy$, $\bar{z}=x-iy$,

$$U_z = \frac{1}{2}\left(\frac{\partial U}{\partial x} - i\,\frac{\partial U}{\partial y}\right), \qquad U_{\bar{z}} = \frac{1}{2}\left(\frac{\partial U}{\partial x} + i\,\frac{\partial U}{\partial y}\right),$$

$$U_{z\bar{z}} = \frac{1}{4}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) = \frac{\Delta U}{4},$$

and where the coefficients $a(z, \bar{z})$, $b(z, \bar{z})$ and $c(z, \bar{z})$ are analytic functions of both variables z and $\bar{z}$. The equation (1.1) is equivalent to the system of two real equations

$$\begin{aligned} \Delta U^{(1)} + 2AU_x^{(1)} + 2BU_y^{(1)} + 2CU_x^{(2)} + 2DU_y^{(2)} & \\ + 4c_1U^{(1)} - 4c_2U^{(2)} &= 0, \\ \Delta U^{(2)} - 2CU_x^{(1)} - 2DU_y^{(1)} + 2AU_x^{(2)} + 2BU_y^{(2)} & \\ + 4c_2U^{(1)} + 4c_1U^{(2)} &= 0, \end{aligned}$$

where

$$\begin{aligned} &U=U^{(1)}+iU^{(2)}; \quad 2A=(a+\bar{a})+(b+\bar{b}); \quad 2B=i[(\bar{a}-a)-(\bar{b}-b)]; \\ &c=c_1+ic_2; \qquad\quad 2D=(a+\bar{a})-(b+\bar{b}); \quad 2C=i[(a-\bar{a})+(b-\bar{b})]. \end{aligned}$$

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[1] S. Bergman, (a) *Zur Theorie der Funktionen, die eine lineare partielle Differentialgleichung befriedigen*, Rec. Math. (Mat. Sbornik) N.S. vol. 44 (1937) pp. 1169–1198; (b) *The approximation of functions satisfying a linear partial differential equation*, Duke Math. J. vol. 6 (1940) pp. 537–561; (c) *Linear operators in the theory of partial differential equations*, Trans. Amer. Math. Soc. vol. 53 (1943) pp. 130–155; (d) *On the solutions of partial differential equations of the fourth order*, to appear later.