ON THE BERGMAN OPERATORS FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS

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1. Introduction. The frequent occurrence of the partial differential equation

$$L(U) = U_{z\bar{z}} + aU_z + bU_{\bar{z}} + cU = 0, \tag{1}$$

where a, b, c are functions of $z=x+iy$ and $\bar{z}=x-iy$ and $U_z=\partial U/\partial z$, $U_{\bar{z}}=\partial U/\partial\bar{z}$, $U_{z\bar{z}}=\partial^2 U/\partial z\partial\bar{z}$, in the theory of dynamics has led to a considerable amount of investigation. This investigation has in part been directed along the lines of solving boundary value and characteristic value problems and of finding particular solutions. These problems are of course classical questions in the theory of partial differential equations and have been widely discussed in the literature. Among the recent developments on these problems is the work done by Bergman [1–5],[1] who in his investigations has introduced operators

$$P(f) = \int_{-1}^{+1} E(z, \bar{z}, t) f(z[1 - t^2]/2) dt/(1 - t^2)^{1/2} \tag{2}$$

where f is an arbitrary analytic function of one complex variable. These operators shall be called the Bergman operators. They transform the class A of analytic functions of one complex variable into a certain class of functions $C(E)$, which has the following properties:

(1°) If E is a solution of the equation

$$\begin{aligned} G(E) = (1 - t^2)(E_{\bar{z}t} + aE_t) - t^{-1}(E_{\bar{z}} + aE) \\ + 2zt(E_{z\bar{z}} + aE_z + bE_{\bar{z}} + cE) = 0 \end{aligned} \tag{3}$$

which satisfies certain conditions, then every function U of the class $C(E)$ will be a particular solution of $L(U)=0$ and further there always exist solutions E satisfying the mentioned conditions.

(2°) It is possible to determine two functions, say E_1 and E_2, so that $C(E_1)+C^*(E_2)$ represents the totality of solutions of $L(U)=0$ (see [2, 5]). C^* is an analogous class of functions, the f being an analytic function of $\bar{z}$.

The general problem of finding the Bergman operators for a given partial differential equation (1) has been thoroughly discussed by

Presented to the Society, November 26, 1943; received by the editors April 18, 1943.

[1] The numbers in the brackets refer to the bibliography.