versely if the first row is multiplied by the inverse of c (mod p^k). This inverse exists, and the correspondence is one-to-one, because c is prime to p. This proves (3).

The sum of the probabilities $P_n(ap^\alpha, p^k)$, where a runs through the values $1, 2, \cdots, p^{k-\alpha}$, is clearly the probability that a determinant be divisible by p^α. The terms of this sum can be simplified and collected by use of (3), and we have

$$P_n(0, p^\alpha) = \sum_{r=0}^{k-\alpha} \phi(p^{k-\alpha-r}) P_n(p^{\alpha+r}, p^k). \tag{11}$$

Replacing α by $\alpha+1$, and subtracting the resulting equation from (11), we arrive at (4).

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ON THE NOTION OF THE RING OF QUOTIENTS OF A PRIME IDEAL

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Let $\mathfrak{o}$ be a domain of integrity (that is, a ring with unit element and with no zero divisor not equal to 0), and let $\mathfrak{u}$ be a prime ideal in $\mathfrak{o}$. We can construct two auxiliary rings associated with $\mathfrak{u}$: the factor ring $\mathfrak{o}/\mathfrak{u}$, composed of the residue classes of elements of $\mathfrak{o}$ modulo $\mathfrak{u}$, and the ring of quotients $\mathfrak{o}_\mathfrak{u}$, composed of the fractions whose numerator and denominator belong to $\mathfrak{o}$, but whose denominators do not belong to $\mathfrak{u}$. These constructions are of paramount importance in algebraic geometry; if $\mathfrak{o}$ is the ring of a variety V, there corresponds to $\mathfrak{u}$ a subvariety U of V; $\mathfrak{o}/\mathfrak{u}$ is the ring of U, whereas the ring $\mathfrak{o}_\mathfrak{u}$ is the proper algebraic tool to investigate the neighborhood of U with respect to V.

Now, the local theory of algebraic varieties involves the consideration of rings which are not domains of integrity (this, because the completion of a local ring may introduce zero divisors). Let then $\mathfrak{o}$ be any commutative ring with unit element, and let again $\mathfrak{u}$ be a prime ideal in $\mathfrak{o}$. We may define the factor ring $\mathfrak{o}/\mathfrak{u}$ exactly in the same way as above, but we cannot so easily generalize the notion of the ring of quotients $\mathfrak{o}_\mathfrak{u}$. If there exist zero divisors outside $\mathfrak{u}$, these zero divisors cannot be used as denominators of fractions, which shows that the definition of $\mathfrak{o}_\mathfrak{u}$ cannot be extended verbatim. If we

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