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THE SYMMETRIC JOIN OF A COMPLEX

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1. **The definition of J .** Let K be a polyhedron. With each pair of distinct points p, q of K we associate a closed line segment pq . No distinction is made between p and q and the corresponding end points of pq . The length of pq is a continuous function of p and q , and the length approaches zero if p and q approach a common limit. Distinct segments do not intersect except at a common end point. The points of these segments with their obvious natural topology make up J , the symmetric join of K . This space arises in [4]¹ in connection with the problem of finding the chords of a manifold that are orthogonal to the manifold.

2. **The subdivision of J .** Let the mid-point of pq be denoted by $\Delta p \times q = \Delta q \times p$, and let $p = \Delta p \times p$. These points $\Delta p \times q$ make up the symmetric product S of K . Let the mid-point of the segment from p to $\Delta p \times q$ be denoted by $p \times q$, and let $p = p \times p$. These points $p \times q$ make up the topological product $P = K \times K$. Consider the closed segment of pq from $p \times q$ to $q \times p$, it being understood that this segment is the point p when $p = q$. All such segments form the "neighborhood" N_S . Clearly N_S can be homotopically deformed in N_S along the segments pq upon S with S remaining pointwise invariant. Finally consider the closed segment of pq from p to $p \times q$, it being understood that this segment is the point p when $p = q$. All such segments form the "neighborhood" N_K . Clearly N_K can be homotopically deformed in N_K along the segments pq upon K with K remaining pointwise invariant.

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¹ Numbers in brackets refer to the References at the end of the paper.