plication and addition) on indeterminates (which first appear as indeterminate coefficients). The situation is studied from several points of view. I (Abstract point of view): One set of relations may imply another. A set may be contradictory. Examples are given of complete sets; a set is complete when any relation compatible with it is implied by it. II (Realization): Here a linear vector space is considered and linear operators on it which satisfy the same relations as those that are given. The connection with I is given by the fact that relations satisfied in an invariant subspace imply relations in the whole space. In III a ring is considered generated (with the aid of a field) by operators satisfying given relations. The special case when the relations involve multiplication only correspond to a group algebra. IV deals with relations satisfied by operators as a result of their being operators on a vector space of a given dimensionality. (Received October 23, 1943.)

16. H. E. Salzer: *New tables and facts involving sums of four tetrahedral numbers.*

The author has a second empirical theorem about tetrahedral numbers, that is, $(n^3-n)/6$ for integral n. Every tetrahedral number greater than 1 is the sum of four other non-negative tetrahedrals. This theorem has been verified for the first 200 cases in a table expressing every tetrahedral from 4 through 1373701 as a sum of four non-negative tetrahedrals. With the exception of 153, the first 200 triangular numbers $n(n+1)/2$ can each be expressed as the sum of four non-negative tetrahedrals. There are only 45 integers less than or equal to 1000 which cannot be expressed as the sum of four non-negative tetrahedrals. All numbers ending in 0, 5, or 6 which are less than or equal to 2006 are expressible as a sum of four non-negative tetrahedrals. This includes the first 201 cases of each type. It is interesting to note that the smallest example of a number ending in 4 which is not expressible as a sum of four non-negative tetrahedrals occurs at 1314. Thus here is an instance where a statement is true in the first 131 cases, but fails in the 132nd. (Received October 13, 1943.)

17. L. R. Wilcox: *Modularity in Birkhoff lattices.*

The following theorem connecting G. Birkhoff's upper semi-modular lattices with the author's M-symmetric lattices is proved. Let a lattice be called upper semi-modular if $a+b$ covers a, b when a and b cover ab; let a lattice be called M-symmetric if $(a+b)c=a+bc$ for every $a\leqq c$ implies $(d+c)b=d+cb$ for every $d\leqq b$; finally, let a lattice be called of finite dimensional type if every a, b with $a<b$ have a finite principal chain connecting them. Then a lattice of finite dimensional type is upper semi-modular if and only if it is M-symmetric. The purpose of this theorem is to replace the condition of Birkhoff, forceful only when some chain condition is assumed, by a strictly algebraic condition which is suitable for use in the infinite dimensional case. (Received October 19, 1943.)

Analysis

18. Stefan Bergman: *The determination of singularities of functions satisfying a partial differential equation from the coefficients of their series development.*

Let $U(z, \bar{z})=A_{00}+\sum_{m=0}^{\infty}\sum_{m=1}^{\infty}A_{mn}z^m\bar{z}^n$ be a (complex) solution of the equation $L(U)\equiv U_{z\bar{z}}+a_1U_z+a_2U_{\bar{z}}+a_3U=0$ where a_k, $k=1, 2, 3$, are entire functions of two variables $z=x+iy$, $\bar{z}=x-iy$, x, y real. Using the results of the papers Rec. Math.