

BOOK REVIEWS

Differentialgeometrien in den Kugelräumen. Vol. II. *Laguerresche Differentialkugelgeometrie.* By T. Takasu. Tokyo, Maruzen, 1939. 20+444 pp.

This volume is the second of a series of three, in which Professor Takasu attempts to give a unified and systematical representation of his work in the differential geometry of sphere space. Concentrating on the geometries of Möbius, Laguerre, and Lie, he presents in this volume his results in Laguerre geometry. In his exposition he follows as closely as possible the usual pattern of euclidean differential geometry, to the systematic study of which the author has contributed in several previous papers.

Following this scheme of development of his subject, Professor Takasu has divided his field into five parts. The first chapter explains the fundamental elements and the coordinates of Laguerre geometry, and shows how we can obtain a correspondence between euclidean three space and a Laguerre plane by means of minimal (isotropic) projection. The group of congruent transformations in space corresponds, in this transformation, to the Laguerre group of the plane. Then follows, in Chapter II, the Laguerre generalization of plane euclidean theory of curves, which is the theory of Laguerre invariants of oriented circles. In Chapter III we find the analogous theory of oriented spheres, corresponding to space curves in ordinary geometry. The Laguerre analogues of surfaces are congruences of spheres, which are discussed in Chapter IV. The final chapter deals with systems of cones, which correspond to line congruences in ordinary space.

This gives us quite a good picture of the structure and of the content of Laguerre differential geometry. The author has placed all previous theories, as those of Darboux, of Ribaucour, of Study, in their proper place and has enriched his work with the investigations of Blaschke and his school. He has shown how much more there is to Laguerre geometry than we could have expected from the study of even the richest other author, and how his methods allow the systematic penetration of all fields into which ordinary differential geometry has thrown its light. We mention, as examples out of a great many other results, the author's treatment of "*L*-developables" ("*L*-Torsen"), of *L*-minimal surfaces, and of Bonnet's plane coordinates. He also ventures into differential geometry in the large, as in a Laguerre version of the four vertex theorem.

We believe that Professor Takasu would have increased the useful-