

MAPPINGS BY MEANS OF SYSTEMS OF ANALYTIC FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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1. **Introduction.** An analytic mapping of a domain D in the space E_{2k} of k complex variables z_1, \dots, z_k is a mapping defined by

$$(1.1) \quad T: z'_j = f_j(z_1, \dots, z_k), \quad j = 1, \dots, k,$$

where the $f_j(z)$ are analytic in D . We shall consider only univalent (schlicht) domains D contained in the finite portion of the space E_{2k} . An important result in the theory of analytic mappings states that the mapping is topological (that is, 1-1 and bi-continuous) if, and only if, the Jacobian

$$(1.2) \quad J(z) \equiv \partial(f_1, \dots, f_k) / \partial(z_1, \dots, z_k)$$

is different from zero at each point of D (Carathéodory [12]).¹

An analytic mapping T is called an *inner mapping* of a domain D if $TD \subset D$. It is called an *automorphism* of D if T is 1-1 and if $TD = D$.

In 1907 Poincaré [17] showed that, given two domains D and D' , it is not always possible to map D onto D' analytically. Since that time, several general problems have been considered. One of these problems is to indicate some general rules which tell whether or not two given domains can be mapped analytically upon each other. A second problem is to determine a family of special domains, in terms of some simple properties, the family to be such that every domain can be mapped analytically onto one of these special domains.

In this talk I shall deal with a special case of the first of these two problems together with certain results on the second problem. The work on the first problem which will be presented is based upon Henri Cartan's theory of mappings of domains onto domains of circular type. The work given on the second problem is based upon Bergman's theory of representative domains. In preparing this talk, I have used freely the excellent résumé on analytic mapping contained in the book on several complex variables by Behnke and Thullen [4]. I have also used freely material from the manuscript by Bochner and the speaker of a book on several complex variables, now in preparation [10]. I am indebted to Professor Bochner for permission to use

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¹ Numbers in brackets refer to the Bibliography at the end of the paper.