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ON ABEL AND LEBESGUE SUMMABILITY

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1. **Introduction.** A series $\sum_1^\infty a_n$ is called Abel summable to the value s if the power series $\sum a_n r^n$ converges for $0<r<1$, and if $\sum a_n r^n \to s$ as $r \uparrow 1$; it is called Lebesgue summable if the sine series

$$\sum_1^\infty a_n \frac{\sin nt}{n} = F(t) \tag{1.1}$$

converges in some interval $0<t<\tau$, and if

$$t^{-1}F(t) \to s \quad \text{as} \quad t \downarrow 0. \tag{1.2}$$

We write in the first case $A\sum a_n = s$, and in the latter case $L\sum a_n = s$ (summability A or L respectively). It is known that convergence does not imply L-summability and conversely L-summability does not imply convergence of $\sum a_n$. Tauberian type problems which arise out of this situation have been discussed.[1] It is also known that either convergence or L-summability imply A-summability. As to the converse (restricting ourselves to real a_n) we have proved the following theorems:

THEOREM 1. [8, pp. 582–583]. *If*

$$\sum_n^{2n} (|a_\nu| - a_\nu) = O(1) \quad \textit{as} \quad n \to \infty, \tag{1.3}$$

and if

$$\sum_1^\infty a_n r^n = O(1) \quad \textit{as} \quad r \uparrow 1, \tag{1.4}$$

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[1] See [8], where further references are given; numbers in brackets refer to the bibliography at the end of this paper.