ON THE CONVERGENCE OF CERTAIN PARTIAL SUMS OF A TAYLOR SERIES WITH GAPS

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We consider the function $f(z)$ determined by the power series

$$f(z) = \sum_{1}^{\infty} c_n z^{\lambda_n} \tag{1}$$

and its direct analytic continuation. For simplicity, it is supposed that $\limsup |c_n|^{1/\lambda_n} = 1$.

We write

$$S_n(z) = \sum_{1}^{n} c_p z^{\lambda_p},$$

$$M(r) = \max_{|z|=r} |f(z)| \qquad (0 < r < 1),$$

$$M(r) = 1 \qquad (r \leqq 0),$$

$$\theta_n = \lambda_{n+1}/\lambda_n - 1.$$

Ostrowski has proved[1] that if $\{\theta_{n_i}\}$ is a sequence extracted from the sequence $\{\theta_n\}$ such that $\liminf \theta_{n_i} > 0$, then every regular point of $f(z)$ on the circle $|z| = 1$ is the center of a circle in which the sequence $\{S_{n_i}(z)\}$ converges uniformly to $f(z)$. Restricting ourselves to the question of convergence at the regular points themselves, we shall prove the following theorem:

If

$$\limsup_{i \to \infty} \frac{\log (M(1 - \theta_{n_i}^2)/\theta_{n_i})}{\lambda_{n_i}\theta_{n_i}^2} < \infty, \tag{2}$$

then $\lim S_{n_i}(z) = f(z)$ *at all regular points of* (1) *on the circle* $|z| = 1$.

For the proof, we shall assume that $\lim \theta_{n_i} = 0$; afterwards, we shall remove this restriction, with the aid of Ostrowski's theorem.

Let z_1 be a regular point for (1) on the circle $|z| = 1$, and let z_0 be a point on the segment joining z_1 to the origin. We write $|z_1 - z_0| = a$, and for every positive integer i we define the three circles

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[1] A. Ostrowski, *Über eine Eigenschaft gewisser Potenzreihen mit unendlich vielen verschwindenden Koefficienten*. Preuss. Akad. Wiss. Sitzungsber. vol. 34 (1921) pp. 557–565. Essentially the same proof is to be found in P. Montel's *Leçons sur les familles normales de fonctions analytiques et leurs applications*, pp. 204–207.