303. Abraham Wald and Jacob Wolfowitz: *An exact test for randomness in the non-parametric case based on serial correlation.*

Let $X_1, \cdots, X_n$ be n chance variables, about the distribution of which nothing is known. Let the problem be to test the (null) hypothesis that $X_1, \cdots, X_n$ are independently distributed with the same distribution function. It is shown that an exact test of this hypothesis based on the serial correlation coefficient can be made. For this purpose the distribution of the serial correlation coefficient in the sub-population consisting of all possible permutations of the observed values is employed. Under the null hypothesis this distribution is independent of the distribution function of $X_i(i=1, \cdots, n)$. Several exact moments are obtained and asymptotic normality is proved. (Received August 7, 1943.)

Topology

304. L. M. Blumenthal: *Metric study of generalized elliptic spaces.* Preliminary report.

Let Σ be semimetric with diameter d, $\phi(x/\rho)$ a real single-valued monotonic decreasing function (ρ, positive parameter) defined over the distance set of Σ, with $\phi(0)=1$, $\phi(d/\rho)=0$. The space is called generalized elliptic $E^{\phi}_{n,\rho}$ provided: I. For each positive integer k and each $k+1$ points $p_1, p_2, \cdots, p_{k+1}$ there corresponds an *allowable matrix* (ϵ_{ij}); $\epsilon_{ii}=1$, $\epsilon_{ij}=\epsilon_{ji}=\pm 1$ $(i, j=1, 2, \cdots, k+1)$ with every nonvanishing principal minor of the determinant $|\epsilon_{ij}\phi(p_ip_j/\rho)|$ positive. II. The integer n is the smallest for which there exist $n+1$ points $p_1, p_2, \cdots, p_{n+1}$ such that $|\epsilon_{ij}\phi(p_ip_j/\rho)|$ does not vanish for any allowable matrix (ϵ_{ij}). For the ordinary elliptic space, Σ is the surface of the sphere $S_{n,\rho}$ with opposite points identified and "shorter arc" metric, while $\phi(x/\rho)\equiv\cos(x/\rho)$. An interesting feature of these spaces is that, in contrast to others (that is, euclidean, hyperbolic, spherical) the mutual distances of a set of points does not suffice to determine the dimension of the subspace which contains them. Thus a given set of three numbers may be distances of three points on an $E^{\phi}_{1,\rho}$, and also distances of three points not on any $E^{\phi}_{1,\rho}$, but on an $E_{2,\rho}$. It is found that certain pseudo-$E^{\phi}_{n,\rho}(n+3)$-tuples are contained in an $E^{\phi}_{n+2,\rho}$. New theorems concerning determinants are a by-product of the study. (Received August 3, 1943.)

305. L. M. Blumenthal: *New formulations of some imbedding theorems.*

The theorems deal with congruent imbedding of metric spaces in Hilbert space, and center about the two following results: I. A complete connected ptolemaic metric space in which every point is contained in a closed convex neighborhood is convex. II. A complete, convex, externally convex metric space in which the Theorem of Pythagoras is valid is congruently contained in Hilbert space. An application of a well known theorem of Menger-Schoenberg yields the first result when it is shown that each two points of the space are joined by an arc with everywhere vanishing metric curvature. To establish the second theorem one notes that every pair of lines that intersect at "right angles" is congruently imbeddable in the plane. It follows that the space has the *weak euclidean four-point property* and the conclusion follows from a result due to the writer. (Received August 3, 1943.)