Applied Mathematics

277. Garrett Birkhoff: *The reversibility paradox and camber.*

A comparison is made between the predictions of the (reversible) Kutta-Joukowsky lift theory and the experimental findings of Eiffel, for circular airfoils. It is found that the predicted angle of the chord to the airfoil with the airstream in the position of zero lift ("first axis") varies from two to seven times the observed angle. Whereas with camber 1/7, the observed increase in lift/effective angle of attack is about 83 per cent; the predicted increase 4 per cent. A possible irreversible explanation of this is pointed out. (Received August 14, 1943.)

278. H. B. Curry: *The method of steepest descent for nonlinear minimization problems.*

A practical method for calculating approximately a stationary value of a function $G(x_1, x_2, \cdots, x_n)$ is desirable in connection with certain nonlinear least square problems (abstract 49-11-286). Such a method may be exhibited as follows. Let x_i^0 be an initial approximation. The direction in which $g(t)=G(x_i^0+t\xi_i)$ decreases most rapidly is obtained by putting $\xi_i=-\partial G/\partial x_i$. By trial t_0 can be determined so that $g(t_0)<g(0)$. Then the point $x_i^0+t_0\xi_i$ can be used as a new approximation and the process repeated. It is shown here that if t_0 is chosen so that $g'(t_0)=0$, $g'(t)<0$ for $0\leqq t<t_0$, then the successive approximations converge to a point where $\partial G/\partial x_i=0$ provided that G takes a smaller value at x_i^0 than at any point on the boundary of that region S within which differentiability and other usual conditions hold. Although the process is well known in analysis, it does not appear to have been noticed recently in this connection. It is applicable when the initial approximation x_i^0 is too rough for the standard least square procedure. The problem includes that of solving n simultaneous nonlinear equations in n unknowns which was handled by Cauchy. (Received October 1, 1943.)

279. D. W. Dudley and Hillel Poritsky: *The geometry of cutting and hobbing of worms and gears.*

The exact shape of the teeth of a worm W or a helical gear G which is produced by a milling cutter C or a hob H of a given tooth profile is investigated. This problem and its converse are of great interest in the manufacture of gears and worms. Assuming numerous cutter teeth one may replace the cutter C by a surface of revolution S. In its motion relative to W, S occupies a one-parameter family of positions whose envelope is W. The curve of contact of S_0, any position of S, with the envelope represents C, the curve of deepest cutting. C may be determined from the condition that along it the velocity of the motion of S relative to W is tangent to S. In the converse problem where W is given and S is sought, the motion of W relative to S is utilized in a similar way; this motion can now be simplified to a rotation about the cutter axis. A similar treatment applies to the hob-gear problem except that here a two-parameter family of motions is involved and two kinematic conditions are now required to determine the deepest cutting. (Received August 12, 1943.)

280. Bernard Friedman: *A method of approximating the complex roots of polynomial equations.*

By using successive divisions, an iterative process is set up to approximate the quadratic factors of a polynomial. In this way, the complex roots of largest and small-