THEORY OF INTERSECTION OF TWO PLANE CURVES

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1. **Introduction.** This paper is concerned with the study of the projective differential geometry of two plane curves intersecting at an ordinary point. The method used is similar to the one given by the author[1] in a previous paper.

In §2 we show the existence of two projective invariants I, J determined respectively by the fourth order of the two curves at the point of intersection.

In §3 by introducing a certain covariant triangle of reference associated to the two curves at the point of intersection, we reduce the invariants I, J to simpler form and give them geometrical characterizations.

Finally, in §4 a covariant point is suitably chosen for the unit point of the coordinate system in order to obtain the canonical power series expansions of the two curves at the point of intersection. With reference to the vanishing or nonvanishing of the two invariants I, J we have four different types of expansions. The absolute invariants in the expansions of each type are interpreted geometrically in terms of certain double ratios.

2. **Derivation of invariants.** Let us establish a projective coordinate system in a plane, in which a point has nonhomogeneous coordinates x, y and homogeneous coordinates x_1, x_2, x_3, connected by the relations $x=x_2/x_1$, $y=x_3/x_1$. The context will show in any instance which coordinates are being used. Then let us consider two curves C, $\overline{C}$ in the plane which intersect at an ordinary point O with distinct tangents t, $\bar{t}$. If we choose the point O to be the vertex (1, 0, 0) of the triangle of reference, and the tangents t, $\bar{t}$ to be the sides $y=0$, $x=0$ of the triangle, then the power series expansions of the curves C, $\overline{C}$ in the neighborhood of the point O may be written in the form

$$(1)\qquad \begin{cases} C: & y = a_1x^2 + a_2x^3 + a_3x^4 + a_4x^5 + \cdots , \\ \overline{C}: & x = \bar{a}_1y^2 + \bar{a}_2y^3 + \bar{a}_3y^4 + \bar{a}_4y^5 + \cdots , \qquad a_1\bar{a}_1 \neq 0. \end{cases}$$

Let us now make a most general transformation of coordinates which shall leave the point O and the tangents t, $\bar{t}$ unchanged. This

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[1] C. C. Hsiung, *Projective differential geometry of a pair of plane curves*, to appear in the Duke Mathematical Journal.