MINIMAL A-SETS, INFINITE ORBITS, AND FIXED ELEMENTS

G. E. SCHWEIGERT

Throughout this note S denotes a semi-locally-connected continuum[1] and $T(S)=S$ an onto-homeomorphism. If E is a set of S such that $T^k(E)=E$ for some positive integer k, then E has a *finite period*, otherwise the period for E is *infinite*. The set of all images of E under T and its inverse T^{-1} is said to be the *orbit* of E. If E is of period $k=1$, then E is *invariant* under T. We investigate the least invariant A-set which contains the orbit of a cyclic element E, when the period and orbit for E are infinite. This work occupies an intermediate position between a previous paper,[2] wherein certain general results are directed toward the study of finite orbits, and the problem of the action of $T(S)=S$ in general. We follow here the spirit of the work of Ayres begun in his paper, *On transformations having periodic properties*, Fund. Math. vol. 33 (1939) pp. 95–103. With Ayres we denote the unique cyclic chain between two cyclic elements E and D by $C(E, D)$.

THEOREM[3] A. *If E is any cyclic element of S with an infinite period and B is the least (invariant[4]) A-set containing the orbit of E then one of the following cases must occur:*

(a) *B contains exactly one fixed element F. In this case if B is cyclic then E is a single point which is a cut point of S lying in B. The set $S-B$ then has infinitely many distinct components bounded by images of E. (Of course it may also have other components not bounded by these images.) If B is not cyclic then for any element E' of the orbit of E the set B is the closure of the orbit of the cyclic chain $C(E', F)$.*

(b) *B contains exactly two fixed elements X and Y. In this case X*

Presented to the Society, April 23, 1943; received by the editors March 15, 1943.

[1] See G. T. Whyburn, *Analytic topology*, Amer. Math. Soc. Colloquium Publications vol. 28, 1942, pp. 64–98. It is assumed that the reader is familiar with the cyclic element theory and the general terminology of this publication. We refer to this book hereafter as ATW with the numbers for theorems in parentheses.

[2] *Fixed elements and periodic types for homeomorphism on s.l.c. continua*, Proc. Nat. Acad. Sci. U. S. A. vol. 29 (1942) p. 52. To appear in full in the Amer. J. Math. under the same title. This paper will be referred to as F; the numbers in parentheses refer to theorems of the complete paper.

[3] Compare F(2.4). Each theorem of this paper has as a corollary the special case in which S is a dendrite. The form used in stating Theorem A and the general nature of the proof are at the suggestion of the referee.

[4] The least A-set containing an invariant set is invariant.