SPECTRAL THEORY

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Spectral theory has for its origin the classical theory of finite matrices and, in a sense, includes all theories which may be specialized to some particular phase of that classical theory. Such a meaning for the term spectral theory is perhaps too broad, for its acceptance would force one, as is becoming increasingly evident, to admit that spectral theory embraces not a small part of such fields of mathematics as the ergodic theory, the theory of probability, absolutely convergent trigonometric series, Fourier transforms, the theory of continuous functions on a bicompact Hausdorff space, the theory of partially ordered rings and Abelian groups, and perhaps even the well known embedding theorem concerning completely regular topological spaces. In fact, even though by far the greatest part of spectral theory that has been at present developed has for its roots the properties of a very special type of matrix, namely the Hermitian matrix, it is certainly true that modern developments have made significant contacts with all the theories just mentioned.

Although I shall touch briefly upon certain features of these modern developments my chief concern here will not be an attempt to describe the wealth of ideas that have grown out of the properties of an Hermitian matrix, but rather to show how the properties of a general matrix can guide the way to the solution of a special class of problems of vital interest in analysis. The type of problem with which I shall be concerned is illustrated by the following question. How can it be determined whether or not a given sequence of polynomials $f_n(T)$ in a linear operator T on a Banach space converges to a specified type of limit operator? Before making this problem more precise I should like to show how the elementary properties of a finite matrix can, if put in the proper form, give an immediate answer to all such questions of convergence for a finite dimensional space.

Suppose $\mathfrak{X}$ is an m-dimensional linear vector space over the field of complex numbers (or over any algebraically closed field). Let T be a linear operator in $\mathfrak{X}$, that is, T is an $m\times m$ matrix whose elements are complex numbers. For every complex number λ and every

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The address by Taylor which follows this was delivered on the same day before the Stanford meeting. To some extent the two papers overlap each other in subject matter, but they also complement each other through the divergence of the authors' points of view.