THE STRONGER FORM OF CAUCHY'S INTEGRAL THEOREM

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1. Introduction. The so-called weaker and stronger forms[1] of Cauchy's integral theorem are the following.

THEOREM 1. CAUCHY'S INTEGRAL THEOREM (WEAKER FORM). *If* $f(z)$ *is holomorphic on the finite simply-connected open domain* D, *and if* C *is a closed rectifiable curve in* D, *then*

$$\int_C f(z)dz = 0.$$

THEOREM 2. CAUCHY'S INTEGRAL THEOREM (STRONGER FORM). *If* $f(z)$ *is holomorphic on the interior* D *of a simply closed rectifiable curve* C, *and continuous on* $D+C$, *then*

$$\int_C f(z)dz = 0. \tag{1}$$

Each of these theorems follows readily when it has been established that there is a sequence of closed curves $\{C_n\}$ in D, of uniformly bounded length, converging in the sense of Fréchet to C, such that

$$\int_{C_n} f(z)dz = 0.$$

In the case of Theorem 2 it appears to be more difficult to establish the existence of the sequence $\{C_n\}$, since the convergence must be from only one side; but the difficulty may be overcome by more or less tedious topological considerations, a program which has been undertaken by a number of authors.[2] Recent excellent proofs by Reid and Hestenes, loc. cit., appear to be about as simple as a valid elementary proof of this result could be.

The above type of proof applies equally well to yield the corresponding stronger form of Green's lemma, as some authors have

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[1] This terminology is taken from M. H. A. Newman, *Elements of the topology of plane sets of points*, Cambridge, England, 1939, pp. 154, 156.

[2] For bibliographical discussions see S. Pollard, *On the conditions for Cauchy's theorem*, Proc. London Math. Soc. vol. 21 (1923) pp. 456–482; E. Kampe, *Zu dem Integralsatz von Cauchy*, Math. Zeit. vol. 35 (1932) pp. 539–543; W. T. Reid, *Green's lemma and related results*, Amer. J. Math. vol. 63 (1941) pp. 563–574; M. R. Hestenes, *An analogue of Green's theorem in the calculus of variations*, Duke Math. J. vol. 8 (1941) pp. 300–311.