ON NON-CUT SETS OF LOCALLY CONNECTED CONTINUA

W. M. KINCAID

W. L. Ayres[1] and H. M. Gehman[2] have proved independently that if a locally connected continuum S contains a non-cut point p, there exists an arbitrarily small region R containing p and such that $S-R$ is connected. Our paper is concerned with certain generalizations of this theorem.

We shall consider a space S which is a locally connected continuum and contains a closed set P such that $S-P$ is connected. We show that under these hypotheses P can be enclosed in an open set R, the sum of a finite number of regions, whose complement is a locally connected continuum. We show further that if there exists a family of sets $\mathfrak{F}$ no element of which separates $S-P$, then there exist two open sets R and R' (with $R \supset R' \supset P$) of the above type and having the property that no element of $\mathfrak{F}$ contained in $S-R$ separates $S-R'$. When the elements of $\mathfrak{F}$ are single points, it is possible to choose $R'=R$; but this is not possible in the more general case.

We close by showing that if S is not separated by any element of $\mathfrak{F}$ plus any set of n points, and if Q is the sum of n sets of sufficiently small diameter and having sufficiently great mutual distances, then the set $S-Q$ has at most one component whose diameter is greater than a preassigned positive quantity, and this component is not separated by any element of $\mathfrak{F}$ at a sufficiently great distance from Q.

We recall some well known results.[3]

Let M be a locally connected continuum. Then:

(1) *M is a metric space having property S.*[4]

(2) *M is the sum of a finite number of arbitrarily small connected*

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[1] See W. L. Ayres, *On continua which are disconnected by the omission of a point and some related problems*, Monatshefte für Mathematik und Physik vol. 36 (1929) pp. 135–147. The theorem quoted here corresponds to Theorem 2 p. 149.

[2] See H. M. Gehman, *Concerning certain types of non-cut points, with an application to continuous curves*, Proc. Nat. Acad. Sci. U.S.A. vol. 14 (1928) pp. 431–433. Theorem 4 p. 432 is essentially that quoted here.

[3] See G. T. Whyburn, *Analytic topology*, Amer. Math. Soc. Colloquium Publications, vol. 28 (1942) p. 20 ff.

[4] A set is said to have property S if for any $\epsilon>0$ it can be expressed as the sum of a finite number of connected sets of diameter less than ϵ. The property was first introduced by W. Sierpinski in his paper *Sur une condition pour qu'un continu soit une courbe jordanienne*, Fund. Math. vol. 1 (1920) pp. 44–60.