SET-COORDINATES FOR LATTICES[1]

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On p. 26 of Garrett Birkhoff's *Lattice theory* (Amer. Math. Soc. Colloquium Publications, vol. 25, 1939) we find the following theorem.

THEOREM 2.12. *Any partially ordered system has a one-one representation by sets which preserves inclusion and meets.*

In the proof of the above theorem each element of the partially ordered system is represented by its "normal hull." We shall call this the *regular* representation.

DEFINITION 1. *By a representation by sets of the elements of a lattice L we mean any one-one representation by sets which preserves inclusion and carries meets into set-products.*

The above-mentioned regular representation is by no means the most economical. The following illustration bears out this point by using ten elements instead of the twenty-eight required by the regular representation.

ILLUSTRATION 1. In the following manner we can represent the lattice in Figure 5 on p. 49 of Birkhoff (loc. cit.) by suitable set-coordinates (using sets of integers):

$O=(\ )$, $v_3=(1)$, $v_2=(2)$, $v_1=(3)$,
$d_1=(1, 2)$, $d_2=(1, 3)$, $d_3=(2, 3)$,
$a_1=(1, 2, 4)$, $a_2=(1, 3, 5)$, $d=(1, 2, 3)$, $a_3=(2, 3, 6)$,
$x_1=(1, 2, 4, 7)$, $x_2=(1, 3, 5, 8)$, $e_1=(1, 2, 3, 4)$, $e_2=(1, 2, 3, 5)$, $e_3=(1, 2, 3, 6)$, $x_3=(2, 3, 6, 9)$,
$b_1=(1, 2, 3, 4, 7)$, $b_2=(1, 2, 3, 5, 8)$,
$c=(1, 2, 3, 4, 5, 6)$, $b_3=(1, 2, 3, 6, 9)$,
$c_1=(1, 2, 3, 4, 5, 6, 7)$, $c_2=(1, 2, 3, 4, 5, 6, 8)$, $c_3=(1, 2, 3, 4, 5, 6, 9)$,
$u_3=(1, 2, 3, 4, 5, 6, 7, 8)$, $u_2=(1, 2, 3, 4, 5, 6, 7, 9)$, $u_1=(1, 2, 3, 4, 5, 6, 8, 9)$,
$I=(1, 2, 3, 4, 5, 6, 7, 8, 9)$.

We remark that only nine elements (the integers 1, 2, 3, 4, 5, 6, 7, 8, 9) together with certain of the "sums" of these integers are necessary to represent this lattice (instead of the twenty-eight elements in the given Hasse diagram required in the regular representation).

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