Applied Mathematics

155. H. R. Branson: *On the difference equation of a general quantum mechanical problem.*

The problems of quantum mechanics may be derived from the classical functions either in the q-language or the p-language. In the q-language $F(q, p) = F(q, hi^{-1}\partial/\partial q)$; in the p-language $F(q, p) = F(-hi^{-1}\partial/\partial p, p)$. With certain types of potential function, the p-language leads to difference equations. Because of the great popularity of quantum mechanics the expression of a general problem in difference equations may aid in "the creation of a usable mathematics of the discrete". The usual equation in one dimension is $H(q, p)\psi = E\psi$ where $H(q, p) = (1/2m)p^2 + V(q)$. The equation becomes in the p-language $(1/2m)p^2\psi + V(-hi^{-1}\partial/\partial p)\psi = E\psi$. Limiting potentials $V(q)$ to those which may be expanded in Fourier series, it follows that: $V(-hi^{-1}\partial/\partial p)\psi = \sum_{k=0}^{\infty}[a_k\psi(p+kh) + b_k\psi(p-kh)]$. With some obvious simplification, the general difference equation for this type of one dimensional quantum mechanical problem may be written in the form $\sum_{k=-\infty}^{\infty} C_k\psi(p+kh) = 0$, wherein all C_i are constants except $C_0 = (p^2/2m + a_0 + b_0 - E)$. Some examples and extensions are discussed. (Received March 25, 1943.)

156. A. H. Fox: *Integral representation of the flow of a compressible fluid around a cylinder.* II.

The method outlined in Part I (Bull. Amer. Math. Soc. abstract 49-1-60) is applied for values of $q=0$ and $q=1/2$ to the flow of a perfect gas around a circular obstacle. The nature of the variations in the boundary of the flow are discussed, and the effect of compressibility on the pressure is considered. (Received February 22, 1943.)

157. J. F. Harding and Isaac Opatowski: *An approximate formula for the Legendre elliptic integral of the second kind.*

By means of ultraspherical polynomials $E = \int_0^{\pi/2}(1-k^2\sin^2\phi)^{1/2}d\phi$ is expanded in $E = (1+k')\sum a_n k_1^n$ where $k' = (1-k^2)^{1/2}$, $k_1 = (1-k')/(1+k')$. By a suitable change of the series $\sum a_n k_1^n$ the formula $E = (\pi/4)(1+k')[0.75 + 0.1875k_1^2 + (4-k_1^2)^{-1}] + \epsilon$ is obtained, where $0 \leqq \epsilon \leqq 1 - (61\pi/192) = 0.00189 \cdots$, which is a simpler and a closer approximation than the formulas of G. A. Grünberg (Applied Mathematics and Mechanics vol. 1 (1933) pp. 61–69) or those given by J. Thomae (*Formeln und Sätze aus dem Gebiete der elliptischen Funktionen*, Leipzig, 1905, p. 29) or by W. Laska (Sammlung von Formeln, pp. 501–502). (Received March 26, 1943.)

158. Wilfred Kaplan and Max Dresden: *Topology of the molecular N-body problem.*

The N molecules of a gas are considered as mass-particles exerting forces on each other derived from a potential of the form $ar^{-n} - br^{-m}$, $n > m > 0$, $a > 0$, $b > 0$. The force is thus highly repulsive for small r and weakly attractive for large r. The energy integral: Total Kinetic Energy plus Total Potential Energy = Const. $\equiv C$ is then interpreted as restricting the trajectories of the system, in the corresponding $6N$-dimensional phase space, to a $(6N-1)$-dimensional hypersurface $M(C)$. It is shown that for $C \geqq 0$, $M(C)$ has the topological structure of a $(6N-1)$-sphere, minus certain $(6N-4)$-spheres corresponding to collisions. For $-\delta < C < 0$, where δ is a certain explicitly known function of a, b, n, m (not of N), $M(C)$ is homeomorphic to the topo-