SOME PROPERTIES OF MEASURABLE FUNCTIONS

H. FEDERER AND A. P. MORSE

1. **Introduction.** Throughout this paper the letter I will denote some fixed closed interval and f a numerically valued measurable function on I. It is our purpose to establish certain general properties of f. We point out in §4 that two theorems of Banach are almost immediate consequences of these properties. We suspect that further use can be made of our results.

2. **Some notations.** We define

$$X^\wedge = \underset{y}{E}\,[y = f(x) \text{ for some } x \in X], \qquad X \subset I,$$

$$Y^\vee = \underset{x}{E}\,[f(x) \in Y], \qquad Y \subset I^\wedge.$$

Writing $X^{\wedge\vee} = (X^\wedge)^\vee$ and $Y^{\vee\wedge} = (Y^\vee)^\wedge$ we note that the relations

$$X \subset X^{\wedge\vee}, \qquad Y = Y^{\vee\wedge},$$

$$\left(\sum_{n=1}^{\infty} X_n\right)^\wedge = \sum_{n=1}^{\infty} X_n^\wedge, \qquad \left(\sum_{n=1}^{\infty} Y_n\right)^\vee = \sum_{n=1}^{\infty} Y_n^\vee,$$

$$\left(\prod_{n=1}^{\infty} X_n\right)^\wedge \subset \prod_{n=1}^{\infty} X_n^\wedge, \qquad \left(\prod_{n=1}^{\infty} Y_n\right)^\vee = \prod_{n=1}^{\infty} Y_n^\vee,$$

$$X_1^\wedge - X_2^\wedge \subset (X_1 - X_2)^\wedge, \qquad Y_1^\vee - Y_2^\vee = (Y_1 - Y_2)^\vee,$$

$$YX^\wedge = (Y^\vee X)^\wedge,$$

hold whenever $X, X_1, X_2, \cdots$ are subsets of I and $Y, Y_1, Y_2, \cdots$ are subset of $I^\wedge$.

We further define

$$\{y\} = \underset{z}{E}\,[z = y],$$

$$\mathfrak{F} = \underset{y}{E}\,[\{y\}^\vee \text{ is finite}],$$

$$\mathfrak{K} = \underset{y}{E}\,[\{y\}^\vee \text{ has at least } \aleph_0 \text{ elements}],$$

$$\mathfrak{Q} = \underset{y}{E}\,[\{y\}^\vee \text{ has more than } \aleph_0 \text{ elements}].$$

Received by the editors June 4, 1942.