ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

103. A. A. Albert: *Algebras derived by non-associative matrix multiplication.*

Let $\mathfrak{C}$ be any J-involutorial algebra and define its isotopes $\mathfrak{C}_\rho$, $\mathfrak{C}_\kappa$, $\mathfrak{C}_{\kappa\rho}$ by $x, y = x(yJ)$, $(x, y) = (xJ)y$, $[x, y] = (xJ)(yJ)$, respectively. When $\mathfrak{C}$ has a unity quantity the algebras $\mathfrak{C}_\rho$, $\mathfrak{C}_\kappa$, $\mathfrak{C}_{\kappa\rho}$ are all semi-simple if and only if $\mathfrak{C}$ is semi-simple, they are simple if and only if $\mathfrak{C}$ is either simple or a direct sum $\mathfrak{S} \oplus \mathfrak{S}J$ where $\mathfrak{S}$ is simple. If $\mathfrak{N}$ is the radical of $\mathfrak{C}$ then $\mathfrak{N} = \mathfrak{N}J$ forms the radical of the isotopes. The structure of row algebras, that is, subalgebras of $\mathfrak{C}_\rho$ is determined. In particular it is shown that there exist real linear spaces of matrices forming algebras under row by row but not under row by column multiplication. They are never semi-simple. (Received January 26, 1943.)

104. Joseph Bowden: *The quaternary permutation function and a generalization of Newton's binomial theorem and Vandermonde's permutation theorem.*

If a and d are any finite numbers, m any integer, r a primary number and ρ an integer whose elements are the primary numbers r_1 and r_2, define $(a, d)P(m, r) = \prod_{k=m+1}^{m+r}(a-(k-1)d)$ and $(a, d)P(m, \rho) = (a, d)P(m-r_2, r_1) : (a, d)P(m-r_2, r_2)$. From these definitions it follows that $(a, d)P(m, 0) = 1$ and $(a, d)P(m, \rho) = : [(a, d)P(m+\rho, -\rho)]$. The operation P is quaternary because it acts on four operands. From the quaternary permutation function $(a, d)P(m, \rho)$ by putting $d=0$ it is found that $(a, d)P(m, \rho) = a^\rho$. By putting $d=1$ or $a=\rho d$ or $m=0$ three ternary functions are obtained. By making two of these three substitutions three binary functions are obtained. In particular if $d=1$ and $m=0$, $(a, d)P(m, \rho) = aP\rho$, the binary permutation function. By making all three of these substitutions $(\rho, 1)P \cdot (0, \rho) = \rho!$ is obtained, the unary factorial function. As examples, it is found that $0! = 1$ and $\rho! = \infty$ if ρ is negative. By mathematical induction the following theorem is proved, of which Newton's binomial theorem and Vandermonde's permutation theorem are special cases: If r is a primary number or zero, a, b, d any finite numbers, except that, if r and d are both zero, neither a nor b nor $a+b$ is zero, and m and n are any integers, then $(a+b, d)P(m+n, r) = \sum_{k=1}^{r+1} rC(k-1) \cdot (a, d)P(m, r-(k-1)) \cdot (b, d)P(n, k-1)$. By putting $d=0$, $(a+b)^r = \sum_{k=1}^{r+1} rC(k-1) \cdot a^{r-(k-1)} \cdot b^{k-1}$, which is Newton's theorem. By putting $d=1$ and $m=0$ $(a+b)Pr = \sum_{k=1}^{r+1} rC(k-1) \cdot aP(r-(k-1)) \cdot bP(k-1)$, which is Vandermonde's theorem. (Received February 1, 1943.)