BOOK REVIEWS

Algebraic topology. By Solomon Lefschetz. (American Mathematical Society Colloquium Publications, vol. 27.) New York, American Mathematical Society, 1942. 6+389 pp. $6.00.

Since the publication of Lefschetz's *Topology* (Amer. Math. Soc. Colloquium Publications, vol. 12, 1930; referred to below as (L)) three major advances have influenced algebraic topology: the development of an abstract complex independent of the geometric simplex, the Pontrjagin duality theorem for abelian topological groups, and the method of Čech for treating the homology theory of topological spaces by systems of "nerves" each of which is an abstract complex. The results of (L), very materially added to both by incorporation of subsequent published work and by new theorems of the author's, are here completely recast and unified in terms of these new techniques. A high degree of generality is postulated from the outset. The abstract point of view with its concomitant formalism permits succinct, precise presentation of definitions and proofs. Examples are sparingly given, mostly of a simple kind, which, as they do not partake of the scope of the corresponding text, should be intelligible to an elementary student. But this is primarily a book for the mature reader, in which he can find the theorems of algebraic topology welded into a logically coherent whole.

The first chapter presents the set-theoretic considerations which will underlie both the spaces to be studied later and the algebraic machinery used to study them. Topological spaces are defined as point sets in which open sets are specified subject to the three usual axioms. Mappings (that is, continuous single-valued transformations) are next defined so that their properties may be developed along with those of the space. By using topological products of open or of closed line segments, Euclidean n-space, the n-cell and the Hilbert parallelotope are introduced. A space is called *compact* if for every covering $\{U_a\}$ by open sets there exists a finite subset of $\{U_a\}$ which covers the space. This is the property often called "bicompactness." The principal consequences of the separation axioms particularly for compact spaces precede a definition of *normality* and *characteristic function* and the Tychonoff imbedding theorem, which, via metric spaces, leads to the Urysohn metrization theorems. A set is a *directed system* if there is a relation $>$ between certain pairs a,b of its elements such that $c>b$ and $b>a$ implies $c>a$, and for every pair a,b of elements there is an element c such that $c>a$ and $c>b$. Using these, inverse