problems associated with the differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \lambda u = 0$$

for various types of boundary conditions when the boundary is rectangular.

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ON THE CONVERGENCE OF A CONTINUED FRACTION

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It is known [1] that sufficient conditions for the convergence of the continued fraction

$$b_0 + \frac{a_1}{1} + \frac{a_2}{1} + \cdots, \tag{1}$$

where the elements are complex numbers, are

$$| a_2 | \geqq 5, \quad | a_{2n} | \geqq 25/4, \quad | a_{2n-1} | \leqq 1/4, \quad n = 2, 3, 4, \cdots. \tag{2}$$

The purpose of this note is to extend this result.

THEOREM. *If* $|a_{2n+1}| \leqq r \leqq 1/4$ $(n=1, 2, 3, \cdots)$ *and if the numbers* $a_{2n} = \rho_{2n} e^{i\theta_{2n}}$ $(n=1, 2, 3, \cdots)$ *satisfy the conditions*

$$\rho_{2n} \geqq 2(1+r)^2[1 - \cos(\theta_{2n} + \theta_0)], \qquad 0 \leqq \theta_{2n} < \pi - \theta_0, \tag{3}$$

$$\rho_{2n} \geqq 4(1+r)^2, \qquad \pi - \theta_0 \leqq \theta_{2n} \leqq \pi + \theta_0, \tag{4}$$

$$\rho_{2n} \geqq 2(1+r)^2[1 - \cos(\theta_{2n} - \theta_0)], \qquad \pi + \theta_0 < \theta_{2n} \leqq 2\pi, \tag{5}$$

where $\theta_0 = 2$ arc sin r, *the continued fraction* (1) *converges.*

To prove the theorem we employ the continued fraction

$$1 + \frac{x_1}{1} + \frac{x_2}{1} + \cdots \tag{6}$$

where

$$x_{2n} = \frac{(1 + a_{2n-1})(1 + a_{2n+1})}{a_{2n}}, \qquad n = 2, 3, \cdots, \tag{7.1}$$

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