# A MIXED BOUNDARY VALUE PROBLEM

## SOME REMARKS ON A PROBLEM OF A. WEINSTEIN

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At present the unilateral Laplace transform has had many interesting applications. To cite several types we have (a) initial value problems in ordinary differential equations, (b) initial value and boundary value problems in partial differential equations with one space variable, (c) "initial value problems" in ordinary difference equations, and (d) "initial value" and "boundary value problems" in partial difference equations.

Titchmarsh[1] and his collaborators, Cooper[2] and Busbridge,[3] have indicated that much can be done with the finite Laplace transform, that is,

$$\int_a^b e^{-sx}f(x)dx = g(s). \tag{1}$$

The transform (1) contains as a special case a finite Fourier transform, first used by Stokes in 1850 for the solution of certain boundary value problems in mathematical physics. The finite Fourier transform has been recently revived by Doetsch[4] and Kniess.[5] It cannot be applied as widely as (1), since it assumes, a priori, that the boundary value problem naturally has a Fourier series as an expansion. On the other hand (1) slips quite neatly into an expansion which is natural to the boundary value problem.

By applying a transformation of the type (1) to a linear differential equation (ordinary or partial) with constant coefficients under given boundary conditions, boundary functions which are superfluous are introduced. Picone and others have demonstrated that by solving the reduced equation and noting the fact that $g(s)$ is an entire function of the parameter $s$, these superfluous boundary elements may be eliminated. This procedure may become exceedingly difficult to carry through. The method we employ here makes use of a regularity condition which is introduced by rendering the boundary conditions symmetric.

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[1] Titchmarsh, J. London Math. Soc. vol. 14 (1939) p. 118.

[2] Cooper, J. London Math. Soc. vol. 14 (1939) p. 124.

[3] Busbridge, J. London Math. Soc. vol. 14 (1939) p. 128.

[4] Doetsch, Math. Ann. vol. 112 (1935) p. 52.

[5] Kniess, Math. Zeit. vol. 44 (1939) p. 266.