A NOTE ON COMPLEMENTARY SUBSPACES IN A RIEMANNIAN SPACE

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1. Introduction.[2] Let

$$(1.1)\qquad x^\kappa = x^\kappa(u^a), \quad \kappa, \lambda, \mu, \nu = 1, \cdots, n; \; a, b, \cdots, f = 1, \cdots, m,$$

be the equations of a V_m in a V_n with fundamental tensor $g_{\lambda\kappa}$ and let

$$(1.2)\qquad B_a^{\kappa} = \partial_a x^\kappa \equiv \frac{\partial x^\kappa}{\partial u^a}\cdot$$

Then the fundamental tensor and curvature tensor of V_m in V_n are, respectively,

$$(1.3)\qquad 'g_{cb} = g_{\lambda\kappa} B_c^{\lambda} B_b^{\kappa},$$

$$(1.4)\qquad H_{cb}^{\cdot\cdot\kappa} = D_c B_b^{\kappa} \equiv \partial_c B_b^{\kappa} + \Gamma_{\mu\lambda}^{\kappa} B_c^{\mu} B_b^{\lambda} - {}'\Gamma_{cb}^{a} B_a^{\kappa},$$

where D denotes the generalized covariant differentiation with respect to V_m in V_n; and $\Gamma_{\mu\lambda}^{\kappa}$ and $'\Gamma_{cb}^{a}$ are, respectively, the Christoffel symbols of the second kind for V_n and V_m.

By definition a V_m in V_n is said to be *totally semi-umbilical*[3] in V_n if a vector v_κ exists such that

$$(1.5)\qquad v_\kappa H_{cb}^{\cdot\cdot\kappa} = {}'g_{cb}$$

is satisfied at every point of V_m. In particular, this condition is evidently fulfilled when $H_{cb}^{\cdot\cdot\kappa}$ has the form $H_{cb}^{\cdot\cdot\kappa} = {}'g_{cb} n^\kappa$, n^κ being a certain vector; in this case we call V_m *totally umbilical* in V_n.

In what follows we shall consider the subspaces V_m: $x^p = \text{const.}$ in a V_n with fundamental tensor of the form

$$(1.6)\qquad g_{\lambda\kappa} = \begin{pmatrix} g_{cb} & 0 \\ 0 & g_{qp} \end{pmatrix}, \qquad \begin{array}{l} a, b, \cdots, f = 1, \cdots, m, \\ p, q, \cdots, s = m+1, \cdots, n. \end{array}$$

Received by the editors, April 22, 1942.

[1] The author is a Chinese Ying-Keng Funds student. He wishes to thank Professor D. J. Struik for the conversations they had from time to time during his stay at Cambridge, Massachusetts.

[2] For the theory of subspaces V_m in a Riemannian n-space V_n, see Schouten-Struik, *Einführung in der neuern Methoden der Differentialgeometrie* II, Groningen, 1938 chap. 3.

[3] D. Perepelkine, *Sur la courbure et les espaces normaux d'une V_m dans R_n*, Rec. Math. (Mat. Sbornik) N.S. vol. 42 (1935) pp. 81–100.