ing the origin, the theorem from Polya-Szegö may be applied with the $F(z)$ of the theorem taken as $A(z)$. Theorem III(b) then follows immediately.

As an application of Theorem III, let us consider the polynomial $F(z)=\sum_{k=0}^{m}a_kG(k+p)z^k$ where $p>0$ and $G(z)=\Gamma(z)^{-1}=e^{\mu z}\prod_{n=1}^{\infty}(1+n^{-1}z)e^{-z/n}$, the reciprocal of the gamma function. Since $\nu=0$ and all the zeros of $G(z+p)$ are negative, any sector $\omega_1\leqq\arg z\leqq\omega_2\leqq\pi-\omega_1$ containing all the zeros of $A(z)$ will also contain all the zeros of $F(z)$. For example, if $A(z)=(z-2)(z+1-i)$, then $F(z)=0.5z^2-(1+i)z-2+2i$, which has the zeros $(3.058+0.514i)$ and $(-1.058+1.486i)$, both thus being in the sector $0\leqq\arg z\leqq 135°$ containing the zeros of $A(z)$.

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ON THE EXTENSION OF A VECTOR FUNCTION SO AS TO PRESERVE A LIPSCHITZ CONDITION

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1. Introduction. Let V be a two-dimensional Euclidean space, and let x be a vector ranging over V. The vector function $f(x)$ is to be a vector in V defined over a set S of the space V. The Euclidean distance between any two points x and y in the plane is denoted by $|x-y|$. Furthermore $f(x)$ is to satisfy a Lipschitz condition, so that there exists a positive constant K such that

$$(1)\qquad |f(x_1)-f(x_2)|\leqq K|x_1-x_2|$$

holds for all pairs x_1 and x_2 in S.

In event $f(x)$ is a real-valued function of a variable x ranging over a set S of a metric space, then the extension of the definition of $f(x)$ to any set $T\supset S$ so as to satisfy the condition (1) has been accomplished.[1] The present paper establishes the result that the *vector* function $f(x)$ can be extended to any set $T\supset S$ so as to satisfy the Lipschitz condition with the same constant K. In §3 it is shown how the method used to obtain the above result can be applied to yield an extension for the case considered by McShane.[2] If $f(x)$ has its

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[1] E. J. McShane, *Extension of range of functions*, Bull. Amer. Math. Soc. vol. 40 (1934) pp. 837–842.

[2] Loc. cit.