DOUBLE COSET MATRICES AND GROUP CHARACTERS

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1. **Introduction.** The principal theorem of this paper extends to the characters of the irreducible representations of an intransitive group a theorem proved in an earlier paper by the author[1] for the degrees of the irreducible representations of a transitive group. A by-product of the development is the theorem that the sum of the traces of the permutations of any subgroup of a permutation group is not less than the corresponding sum for one of its cosets.

Every finite permutation group G, of order g and degree n, can be written as a group of permutation matrices. The $n \times n$ matrix $R(\gamma)$ which corresponds to the element γ of G can be written as a direct sum of submatrices $R^t(\gamma)$ of n^t dimensions corresponding to the n^t symbols of a transitive constituent C^t of G.[2] Associated with such a transitive constituent C^t is a class of conjugate subgroups, $H_\tau^t = (\gamma_\tau^t)^{-1} H^t \gamma_\tau^t$, each of order h^t, of which H^t shall be the subgroup leaving fixed the first symbol of C^t, and H_τ^t the subgroup leaving fixed the τth symbol. If γ_α is any element of G, then in the set of $h^s h^t$ group elements $H^s \gamma_\alpha H^t$, each element will appear h_α^{st} times, where h_α^{st} is the order of the cross-cut of the subgroups $H_\alpha^s = \gamma_\alpha^{-1} H^s \gamma_\alpha$ and H^t.

Counting each element of the set just once, we define the double coset H_α^{st} by the formula

$$H_\alpha^{st} = H^s \gamma_\alpha H^t / h_\alpha^{st}. \tag{1.1}$$

Any element from a double coset can be chosen as the defining element γ_α. The inverses of the elements of a double coset H_α^{st} themselves form a double coset which we call the inverse double coset and denote by $H_{\alpha'}^{ts}$. The product of two double cosets is a linear combination of double cosets. By considering H_α^{rs} as a sum of h^r/h_α^{rs} left cosets of H^s, and $H_{\beta'}^{st}$ as a sum of $h^t/h_{\beta'}^{st}$ right cosets of H^s, and noting that $H^s H^s = h^s H^s$, it is apparent that each element in the product $H_\alpha^{rs} H_{\beta'}^{st}$ occurs a multiple of h^s times. We define the positive integers $c_{\alpha\beta\eta}^{rst}$ by the formula

$$H_\alpha^{rs} H_{\beta'}^{st} / h^s = \sum_\eta c_{\alpha\beta\eta}^{rst} H_\eta^{rt}. \tag{1.2}$$

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[1] J. S. Frame, *The double cosets of a finite group*, Bull. Amer. Math. Soc. vol. 47 (1941) p. 459.

[2] Throughout this paper the superscripts will refer to the transitive constituents.