Since $\|U_{n_k}(\xi)\| > 0$, the sphere $\|z - U_{n_k}(\xi)\| \leqq \|U_{n_k}(\xi)\|/2$, is non-vacuous. That such a sphere is a p-set was demonstrated in §3. The sphere K we were required to construct has therefore been shown to exist, and Theorem 3 is proved.

LAFAYETTE COLLEGE

ON THE APPROXIMATION OF FUNCTIONS BY SUMS OF ORTHONORMAL FUNCTIONS

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1. **Introduction.** The main object of this paper is to derive, in a simple manner, upper bounds for the norms of the derivative of

$$\sum_{i=0}^{n} a_i\phi_i(x) \tag{1}$$

in C and L^2 spaces, where the a_i are arbitrary constants, and $\{\phi_i(x)\}$ is any set of functions on a given finite or infinite interval (a, b). We apply our method, properly modified, first to the case where the $\phi_i(x)$ are characteristic solutions of conjugate sets of integral equations, then to other classes of functions whose first derivatives $\{\phi_i'(x)\}$ are orthogonal with respect to a weight function $\sigma(x)$. Finally, we apply our results to the question of convergence of sums[1] of type (1) that minimize

$$\int_a^b \rho(x)\left|f(x) - \sum_{i=0}^{n} a_i\phi_i(x)\right|^m dx, \qquad m > 0.$$

The leading results of our investigation may be summarized briefly as follows:

$$\left|\frac{d}{dx}\sum_{i=0}^{n} a_i\phi_i(x)\right| \leqq \lambda_n k(x)\left(\int_a^b [\sum_{i=0}^{n} a_i\phi_i(x)]^2 dx\right)^{1/2} \tag{A}$$

where λ_n is a positive number that increases with n and $k(x)$ is a func-

Presented to the Society, September 5, 1941 under the title *Notes on the approximation of functions by sums of orthonormal functions*; received by the editors February 2, 1942.

[1] For the specialized cases when the approximating functions are trigonometric sums or polynomials, see D. Jackson, *The theory of approximation*, Amer. Math. Soc. Colloquium Publications vol. 11, 1930, pp. 86–89, 96–101.