

vol. 48 (1942) pp. 589–601) which relate tensor algebra and invariant theory are used in the consideration of the elementary problem of the three-line configuration. (Received November 23, 1942.)

16. T. L. Wade: *On conjugate tensors.*

How a contravariant (skew-symmetric) tensor  $V$  of order  $n-p$  may be associated with a covariant skew-symmetric tensor  $U$  of order  $p$ , in an  $n$ -dimensional coordinate system, is well known (see Veblen and von Neumann, *Geometry of complex domains*). This standard association holds only when  $U$  is skew-symmetric. The purpose of this note is to show how a contravariant tensor  $V$ , of defined order and symmetry, can be associated with the covariant tensor  $U$ , where  $U$  is of any type  $[\alpha]$  of symmetry. (Received November 23, 1942.)

17. T. L. Wade: *On the factorization of rank tensors.*

Let  $C_{(i)}^{(o)} = D_{(j)}^{(o)} + E_{(i)}^{(o)}$ , where  $D_{(i)}^{(o)}$  and  $E_{(i)}^{(o)}$  are mutually orthogonal idempotent numerical tensors. Expressions for the contravariant and covariant factors of the rank tensor (see Amer. J. Math. vol. 64 (1942) pp. 725–752) of  $C_{(i)}^{(o)}$  in terms of like factors of the rank tensors of  $D_{(i)}^{(o)}$  and  $E_{(i)}^{(o)}$  are established in this paper. (Received November 23, 1942.)

18. T. L. Wade and R. H. Bruck: *Types of symmetries.*

This paper considers some aspects of symmetries with tensorial significance which are believed not to have appeared in the literature. (Received November 23, 1942.)

19. André Weil: *Differentiation in algebraic number-fields.*

Analogies with function-fields have long ago led E. Noether and others to the conjecture that the theory of the different in number-fields can be built upon some arithmetical analogue of differentiation. This is now done, by defining a derivation modulo an ideal  $\mathfrak{a}$  in a number-field as an operator  $D$  with the following properties: (a)  $D$  maps the ring  $\mathfrak{o}$  of integers in the field into the ring  $\mathfrak{o}/\mathfrak{a}$ ; (b)  $D(\alpha + \beta) = D\alpha + D\beta$ ; (c) if  $\bar{\alpha}, \bar{\beta}$  are the classes of  $\alpha, \beta \pmod{\mathfrak{a}}$ , then  $D(\alpha\beta) = \bar{\alpha} \cdot D\beta + \bar{\beta} \cdot D\alpha$ ;  $D$  is essential if there is  $\alpha$  in  $\mathfrak{o}$ , such that  $D\alpha$  is not a zero-divisor in  $\mathfrak{o}/\mathfrak{a}$ . The different is then the least common multiple of all ideals modulo which there exists an essential derivation. This is easily extended to the relative different, to  $p$ -adic fields, and so on. (Received November 9, 1942.)

20. Alexander Wundheiler: *An algebraic definition of affine space.*

A simple set of axioms for affine geometry based on one operation  $C = hAB$ , where  $h$  is a real number,  $A, B$  points and  $C$  the point collinear with  $A$  and  $B$ , and such that  $CA/CB = h$ , is given. There are essentially five axioms, only one of them involving more than two (namely, three) points. An "affine calculus," which permits the writing of every affine theorem as an implication between formulas, arises from the mentioned operation. (Received November 20, 1942.)

### ANALYSIS

21. R. P. Agnew: *Euler transformations.*

Let  $E(r)$  denote the Euler transformation  $\sigma_n = \sum_{k=0}^n C_{nk} r^k (1-r)^{n-k} s_k$  by means of