This seems to be the generalization of the classical result that a necessary and sufficient condition for the polar components of a matrix $A$ to be commutative is that $A$ be a normal matrix.

REMARKS ON REGULARITY OF METHODS OF SUMMATION

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A doubly infinite matrix $^{1} (a_{mn})$ ($m, n = 1, 2, \cdots$) is said to be regular, if for every sequence $x = \{x_n\}$ with limit $x'$ the corresponding sums $y_m = \sum_{n=1}^{\infty} a_{mn} x_n$ exist for $m = 1, 2, \cdots$, and if $\lim_{m \to \infty} y_m = x'$. An apparently more inclusive definition of regularity is that for each sequence $x$ with limit $x'$ the sums defining $y_m$ shall exist for all $m \geq m_0(x)$ and $\lim_{m \to \infty} y_m = x'$. Tamarkin\(^2\) has shown that $(a_{mn})$ is regular in the latter sense if and only if there exists an $m_1$ independent of $x$ such that the matrix $(a_{mn})$ ($m \geq m_1$, $n \geq 1$) is regular in the former sense. Using point set theory in the Banach space $(c)$, he proves a theorem\(^3\) from which follows the result just mentioned. This note presents an elementary proof of that theorem and discusses some related topics.

THEOREM 1. Suppose the doubly infinite matrix $(a_{mn})$ has the property that for each sequence $x = \{x_n\}$ with limit 0 there exists an $m_0 = m_0(x)$ such that for all $m \geq m_0(x)$, $u_m = \limsup_{k \to \infty} |\sum_{n=1}^{k} a_{mn} x_n| < \infty$. Then there exists an $m_1$ such that $\sum_{n=1}^{\infty} |a_{mn}| < \infty$ for all $m \geq m_1$.

If in addition $\lim_{m \to \infty} u_m = 0$ for each sequence $x$ with limit 0, it will follow\(^4\) that there exists an $N$ such that $\sum_{n=1}^{\infty} |a_{mn}| \leq N < \infty$, for all $m \geq m_1$.

To prove Theorem 1, suppose there were an infinite sequence $m_1 < m_2 < \cdots$ such that $\sum_{n=1}^{\infty} |a_{mn}| = \infty$ for $m \in \{m_r\}$. Let $x_1, \cdots, x_{k_1}$ be chosen with unit moduli and with amplitudes such that

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\(^1\) In this note $a_{mn}$, $x_n$ and $x'$ denote finite complex numbers.


\(^3\) J. D. Tamarkin, loc. cit., p. 242, lines 1–6.