This seems to be the generalization of the classical result that a necessary and sufficient condition for the polar components of a matrix A to be commutative is that A be a normal matrix.

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REMARKS ON REGULARITY OF METHODS OF SUMMATION

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A doubly infinite matrix[1] (a_{mn}) $(m, n=1, 2, \cdots)$ is said to be *regular*, if for every sequence $x=\{x_n\}$ with limit x' the corresponding sums $y_m=\sum_{n=1}^{\infty}a_{mn}x_n$ exist for $m=1, 2, \cdots$, and if $\lim_{m\to\infty} y_m=x'$. An apparently more inclusive definition of regularity is that for each sequence x with limit x' the sums defining y_m shall exist for all $m \geqq m_0(x)$ and $\lim_{m\to\infty} y_m=x'$. Tamarkin[2] has shown that (a_{mn}) is regular in the latter sense if and only if there exists an m_1 *independent of* x such that the matrix (a_{mn}) $(m\geqq m_1,\ n\geqq 1)$ is regular in the former sense. Using point set theory in the Banach space (c), he proves a theorem[3] from which follows the result just mentioned. This note presents an elementary proof of that theorem and discusses some related topics.

THEOREM 1. *Suppose the doubly infinite matrix (a_{mn}) has the property that for each sequence $x=\{x_n\}$ with limit 0 there exists an $m_0=m_0(x)$ such that for all $m\geqq m_0(x)$, $u_m=\lim\sup_{k\to\infty}\left|\sum_{n=1}^{k}a_{mn}x_n\right| < \infty$. Then there exists an m_1 such that $\sum_{n=1}^{\infty}|a_{mn}| < \infty$ for all $m\geqq m_1$.*

If in addition $\lim_{m\to\infty} u_m=0$ for each sequence x with limit 0, it will follow[4] that there exists an N such that $\sum_{n=1}^{\infty}|a_{mn}| \leqq N < \infty$, for all $m\geqq m_1$.

To prove Theorem 1, suppose there were an infinite sequence $m_1<m_2< \cdots$ such that $\sum_{n=1}^{\infty}|a_{mn}| = \infty$ for $m\in\{m_\nu\}$. Let $x_1, \cdots, x_{k_1}$ be chosen with unit moduli and with amplitudes such that

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[1] In this note a_{mn}, x_n and x' denote finite complex numbers.

[2] J. D. Tamarkin, *On the notion of regularity of methods of summation of infinite series*, this Bulletin, vol. 41 (1935), pp. 241–243.

[3] J. D. Tamarkin, loc. cit., p. 242, lines 1–6.

[4] See, for example, I. Schur, *Über lineare Transformationen in der Theorie der unendlichen Reihen*, Journal für die reine und angewandte Mathematik, vol. 151 (1921), pp. 79–111; p. 85, Theorem 4.