surfaces, proving the general Morse relations. The integrals to be made stationary have the form $\iint\{f(X, Y, Z)+k(X^2+Y^2+Z^2)^{1/2}\}\,dudv$ where X, Y, Z are the Jacobians $(y_u, z_u; y_v, z_v)$, and so forth, k is a positive constant, and f satisfies the following four conditions: (1) f is positively homogeneous in X, Y, Z; (2) $\epsilon=f(X, Y, Z)-\{Xf_{X_0}+Yf_{Y_0}+Zf_{Z_0}\}\geqq 0$; (3) $f(X, Y, Z)\geqq m>0$ for $X^2+Y^2+Z^2=1$; and (4) $f(X, Y, Z)<k$ for $X^2+Y^2+Z^2=1$. The first three are usual conditions to be expected; the main restriction is (4) which asserts that the integral is dominantly an area integral. The problem is to find extremal surfaces bounded by a given rectifiable curve. The integral is replaced by $\iint f(X, Y, Z)dudv+kD[x]$ where $D[x]$ is the Dirichlet integral, and the extremal surfaces found are given in isometric representation. (Received August 5, 1942.)

323. J. A. Shohat: *An inequality for definite integrals.* Preliminary report.

Starting with a simple case of the Schwarz inequality, another inequality is derived in a very elementary manner. This inequality seems to be new and may be considered an improvement over the Schwarz inequality. In fact, it is shown that the new inequality yields better results than the Schwarz inequality (it reduces to the latter, in some special cases).—Applications are indicated to orthogonal functions and to polynomials. (Received September 17, 1942.)

Applied Mathematics

324. Lipman Bers and Abe Gelbart: *On solutions of the differential equations of gas dynamics.* I.

By a transformation due to Chaplygin and Busemann the equations of the steady two-dimensional irrotational flow of a gas can be reduced, in the subsonic case, to the form (1): $u_x=\tau(y)v_y$, $u_y=-\tau(y)v_x$, where τ is a given function. Equations of the same form occur in other branches of mechanics of continua. The class of complex-valued functions $f(x+iy)=u+iv$, where u and v satisfy (1) is shown to have many properties in common with analytic functions. Two processes, one the inverse of the other, are introduced: τ-differentiation and τ-integration. The τ-derivative and the τ-integral of f belong to the class. By τ-integrating a complex constant, a_n, n times a function $a_n\cdot Z\cdot{}^n(z)$, of the class is obtained. From these "powers," " polynomials" and "power series" are formed. A "polynomial" of the nth degree possesses n zeros. Any function, f, of the class can be developed in a unique manner in a power series, the "coefficients" a_n, being given by the nth τ-derivatives of f. Domains of convergence, mapping properties, and particular functions are considered. Analogous methods are applied to more general types of equations; in particular to the system $u_x=\tau_1(y)v_y$, $u_y=-\tau_2(y)v_x$ which describes the subsonic as well as supersonic flow. (Received August 27, 1942.)

325. Garrett Birkhoff: *A reversibility paradox in hydrodynamics.*

It is shown that every known differential equation and general boundary condition describing compressible non-viscous flow is reversible, in the sense that it is compatible with reversal of the direction of flow without change in pressure. This is even true in the presence of a conservative force-field. It follows that, contrary to a widespread impression, any theory of non-viscous flow (airfoil theory, resistance to projectiles, or resistance to surface craft in water) which purports to be based entirely on