is a regular Hausdorff mean. By (4.1) we see that $[H, q_n] \supset (C, 1)$. It is easy to show that $(C, 1) \supset [H, q_n]$, thereby proving that $[H, q_n] \approx (C, 1)$. If $c_n = p_n = 1/(n+1)$, then

$$q_n = \frac{1 + 2^{-1} + 3^{-1} + \cdots + n^{-1}}{n+1}$$

and $[H, q_n]$ is a regular Hausdorff mean. This mean does not include $(C, 1)$ inasmuch as $q_n : (n+1)^{-1}$ is unbounded.

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WHIRL-SIMILITUDES, EUCLIDEAN KINEMATICS, AND NON-EUCLIDEAN GEOMETRY

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1. Introduction. The geometry of whirls and whirl-motions in the plane had its origin in a paper by E. Kasner [6],[1] was subsequently developed in a series of papers by Kasner and DeCicco [**3, 7, 8, 9**], adapted to the sphere by Strubecker [**10**], and to 3-space by Feld [**4**]. In this paper we shall, by adjoining three involutory transformations, extend Kasner's whirl-motion group G_6 to a mixed group Γ_6—the *complete whirl-motion group*—composed of eight mutually exclusive, six-parameter families; these families will in turn be extended to seven-parameter families comprising the mixed group Γ_7—the *complete whirl-similitude group*. The principal results obtained are the extension of Kasner's G_6 and two representations of Γ_7: a *kinematic representation on the plane*, §6, and a *representation in quasi-elliptic 3-space*, §7.

2. Slides, turns, and whirls. Let the point of an oriented lineal element E have the rectangular coordinates x, y, and let the inclination of E to the x-axis be the angle θ, $0 \leqq \theta < 2\pi$. Let $z = x + iy$, $\bar{z} = x - iy$, $\zeta = e^{i\theta}$. We shall call z, ζ the *element coordinates* of E (x, y, θ), which, henceforth, shall be represented by the symbol (z, ζ).

DEFINITIONS. *A slide S_s is a lineal element transformation that translates the point of each element along its line the same distance s.*

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[1] The numbers in brackets refer to the bibliography at the end of the paper.