$$0 < t - x < 1/n \text{ implies } [F(t) - F(x)]/(t - x) \leqq n;$$

the remainder of the proof is unaltered. The next lemma is a slight generalization of a theorem of Marcinkiewicz.

LEMMA 5.2. *If $f(x)$ is measurable on $[a, b]$, and has either a left major or a right major, and also has either a left minor or a right minor, then $f(x)$ is Perron integrable on $[a, b]$.*

The proof is that given by Saks, op. cit., p. 253; the principal change is that the reference to his Theorem 10.1 is replaced by a reference to our Lemma 5.1.

Since every P^*-integrable function $f(x)$ is measurable and has right majors and right minors, it is also Perron integrable by Lemma 5.2, and the equivalence of the integrals is established.

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ON THE LEAST PRIMITIVE ROOT OF A PRIME

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It was proved by Vinogradow[1] that the least positive primitive root $g(p)$ of a prime p is $O(2^m p^{1/2} \log p)$ where m denotes the number of different prime factors of $p-1$. In 1930 he[2] improved the previous result to

$$g(p) = O(2^m p^{1/2} \log \log p),$$

or more precisely,

$$g(p) \leqq 2^m \frac{p-1}{\phi(p-1)} p^{1/2}.$$

It is the purpose of this note, by introducing the notion of the average of character sums,[3] to prove that if $h(p)$ denotes the primitive root with the least absolute value, mod p, then

$$| h(p) | < 2^m p^{1/2};$$

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[1] See, Landau, *Vorlesungen über Zahlentheorie*, vol. 2, part 7, chap. 14. The original papers of Vinogradow are not available in China.

[2] Comptes Rendus de l'Académie des Sciences de l'URSS, 1930, pp. 7–11.

[3] The present note may be regarded as an introduction of a method which has numerous applications.