

the place of Study's "Geometrie der Dynamen" in the structure of geometry. Geometric figures have been considered whose tensor representations are: 1. contravariant alternating tensors of valence two; 2. covariant alternating tensors of valence two; 3. mixed tensors of valence two. Among this set are found the affine ancestors of all of the Study figures. Thus the foundation is laid for the tensor interpretation of all of the figures of the "Geometrie der Dynamen." (Received July 30, 1942.)

STATISTICS AND PROBABILITY

282. L. A. Aroian: *The relationship of Fisher's z distribution to Student's t distribution.*

For n_1 and n_2 sufficiently large, $W = (1/\beta)(N/(N+1))^{1/2}z$ is distributed as Student's t with N degrees of freedom, $N = n_1 + n_2 - 1$, $\beta^2 = (1/2)(1/n_1 + 1/n_2)$. If the level of significance is α for Student's distribution, the level of significance for z will be $(N/(N+1))^{1/2}\alpha e^{\beta^2/3-5/12N} < \alpha$. As a corollary it follows that the distribution of z approaches normality, $n_1, n_2 \rightarrow \infty$, with mean zero and variance β^2 . This simplifies a previous proof of the author. Application of this result is made to finding levels of significance of the z distribution. On the whole R. A. Fisher's formulas for finding such levels, n_1 and n_2 large as modified by W. G. Cochran, are superior. The formulas of Fisher-Cochran are compared with the recent formula of E. Paulson. (Received August 1, 1942.)

283. E. J. Gumbel: *Graphical controls based on serial numbers.*

The index m of the observed value x_m ($m=1, 2, \dots, n$) is called its serial number (or rank). A value x of the continuous statistical variable defined by a probability $W(x) = \lambda$ is called a grade (for example, the median). The coordination of serial numbers with grades furnishes two graphical methods for comparing the observations and the theory, namely the equiprobability test based on $m = n\lambda$, and the return periods based on $m = n\lambda + 1/2$. From the distribution of the m th value, determine the most probable serial number $\tilde{m} = n\lambda + \Delta$, where Δ depends upon the distribution. For a symmetrical distribution, the corrections for two grades defined by λ and $1-\lambda$, are $\Delta(1-\lambda) = -\Delta(\lambda)$. For an asymmetrical distribution, calculate the most probable serial number of the mode considered as an m th value. Thus the mode is obtained from the observations, but it is not the most precise m th value. If m is of the order $n/2$ the distribution of the m th value converges towards a normal distribution with a standard deviation $s(x) = (W(x)(1-W(x)))^{1/2}/(w(x)n^{1/2})$. The intervals $x \pm s(x)$ give controls for the equiprobability test, the step function and the return periods. Besides, the standard error of the m th value leads to the precision of a constant obtained from a grade. (Received July 20, 1942.)

284. Mark Kac: *On the average number of roots of a random algebraic equation.*

Let $X_0 + X_1x + \dots + X_{n-1}x^{n-1} = 0$ be an algebraic equation whose coefficients X_0, \dots, X_{n-1} are independent random variables having the same normal distribution with density $\pi^{-1/2} \exp(-u^2)$. If $N_n = N_n(X_0, \dots, X_{n-1})$ denotes the number of real roots of the equation then the average number of roots (mathematical expectation of N_n) is asymptotically equal to $2\pi^{-1} \log n$. Moreover, for $n \geq 2$ the mathematical expectation of N_n is not more than $2\pi^{-1} \log n + 14\pi^{-1}$. This is an improvement of a result of Littlewood and Offord (Journal of the London Mathematical Society,