

( $O$ ) and ( $L$ ) (§I, Theorem 2), the sphere ( $G$ ) having  $G$  for center and orthogonal to ( $Q$ ) (§II, Theorem 1) is the second sphere of anti-similitude of ( $O$ ) and ( $L$ ); hence ( $G$ ) is coaxial with these spheres.

**THEOREM 3.** *The four spheres having for centers the vertices of a tetrahedron and orthogonal to the quasi-polar sphere cut the spheres having for diameters the respective medians of the tetrahedron along four circles belonging to the same sphere, namely, the ( $G$ )-sphere of the tetrahedron.*

The sphere ( $A$ ) having  $A$  for center and orthogonal to the sphere ( $Q$ ) is coaxial with the spheres ( $G$ ) and ( $AG_a$ ), for the centers of these three spheres are collinear and all three are orthogonal to ( $Q$ ). Similarly for the vertices  $B, C, D$  of ( $T$ ).

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## EUCLIDEAN CONCOMITANTS OF THE TERNARY CUBIC

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**1. Introduction; construction of concomitants.** In this paper we use the results of Cramlet [1] and the writer [2] to study the euclidean concomitants of the ternary cubic curve

$$T_{abc}X^aX^bX^c = 0,$$

where  $a, b, c = 1, 2, 3$  and  $T_{abc}$  is symmetric. With tensor algebra as the medium of investigation all types of concomitants are readily constructed, and their geometric interpretations are also readily made in most cases. As is conventional in classical invariant theory, the word concomitant will be used as meaning rational integral concomitant unless stated to the contrary.

As a consequence of Theorem 3 in [2], we have the following theorem.

**THEOREM I.** *Every euclidean concomitant of the ground form  $T_{abc}X^aX^bX^c$  ( $a, b, c = 1, 2, 3$ ) is expressible by composition as a tensor of order zero with the use of the coefficient tensor  $T_{abc}$ , the variable coordinate tensors  $X^a$  and  $U_a$ , and the numerical tensors  $\epsilon^{abc}$ ,  $L_a$ , and  $E^{ab}$ .*

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