A LINEAR TRANSFORMATION WHOSE VARIABLES AND COEFFICIENTS ARE SETS OF POINTS

S. T. SANDERS, JR.

Introduction. While the theory of the linear transformation has been developed in great detail, attention has seldom[1] been called to the transformation T in which variables and coefficients are sets of points. Doubtless the nonexistence of a unique inverse transformation has occasioned this neglect. In this paper the writer studies the iteration of T.

Consider first the transformation

$$T\colon \quad \begin{array}{l} x_1 = a_{11}x_1' + a_{12}x_2' \\ x_2 = a_{21}x_1' + a_{22}x_2' \end{array},$$

whose *set matrix* is

$$M = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix},$$

where the a's and x's are sets of points, and the indicated sums and products refer to set operations. Applying T to the primed variables, we have the product transformation

$$T^2\colon \quad \begin{array}{l} x_1 = \overset{(2)}{a_{11}}\, x_1'' + \overset{(2)}{a_{12}}\, x_2'' \\ x_2 = \overset{(2)}{a_{21}}\, x_1'' + \overset{(2)}{a_{22}}\, x_2'' \end{array},$$

of set matrix

$$M^2 = \begin{Vmatrix} \overset{(2)}{a_{11}} & \overset{(2)}{a_{12}} \\ \overset{(2)}{a_{21}} & \overset{(2)}{a_{22}} \end{Vmatrix},$$

where

$$(1) \qquad \begin{array}{ll} \overset{(2)}{a_{11}} = a_{11} + a_{12}a_{21}, & \overset{(2)}{a_{12}} = a_{11}a_{12} + a_{12}a_{22}, \\ \overset{(2)}{a_{21}} = a_{21}a_{11} + a_{22}a_{21}, & \overset{(2)}{a_{22}} = a_{21}a_{12} + a_{22}. \end{array}$$

Transforming in turn each new set of variables, we obtain product transformations $T^3, T^4, \cdots$, whose set matrices are $M^3, M^4, \cdots$.

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[1] Lowenheim, *Über Transformationen im Gebietekalkül*, Mathematische Annalen, vol. 73 (1913), pp. 245–272; *Gebietsdetermination*, Mathematische Annalen, vol.79 (1919), pp. 223–236.