

NORMAL VARIETIES AND BIRATIONAL CORRESPONDENCES

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1. **Introduction.** As one advances into the general theory of algebraic varieties, one reluctantly but inevitably reaches the conclusion that there does not exist a general theory of birational correspondences. This may sound too reckless a statement or too harsh a criticism, especially if one thinks of the fundamental role which birational transformations are supposed to have in algebraic geometry. Nevertheless our conclusion is in exact agreement with the facts and it is made with constructive rather than with critical intentions. It is true that the geometers have a fairly good intuitive idea of what happens or what may happen to an algebraic variety when it undergoes a birational transformation; but the only thing they know with any certainty is what happens in a thousand and one special cases. All these special cases—and they include all Cremona transformations—are essentially reducible to one special but very important case, namely, the case in which the varieties under consideration are nonsingular (that is, free from singular points). One can give many reasons for regarding as inadequate any theory which has been developed exclusively for nonsingular varieties. One rather obvious reason is that we have as yet no proof that every variety of dimension greater than 3 can be transformed birationally into a nonsingular variety.¹ But there are other, less transient, reasons. Were such a proof available, it would still be advisable to develop the theory of algebraic varieties, *as far as possible*, without restricting oneself to nonsingular projective models. This certainly would be the correct program of work from an arithmetic standpoint. I have a distinct impression that my friends the algebraists have not much use anyway for the resolution of the singularities. All they want is a general uniformization theorem, and now that they have it, they are content.

The following consideration will perhaps carry greater weight with the geometers. It turns out, as I have found out at some cost to myself, that we have to know a lot more about birational correspondences than we know at present before we can even attempt to carry

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¹ The resolution of the singularities of three-dimensional varieties will be carried out in a forthcoming paper of mine.