PSEUDO-CONFORMAL GEOMETRY: FUNCTIONS OF TWO COMPLEX VARIABLES

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1. Introduction. The theory of functions of a *single* complex variable is essentially identical with the conformal geometry of the real (or complex) plane. However, this is *not* the case in the theory of functions of *two* independent complex variables. Any pair of functions of two complex variables induces a correspondence between the points of a real (or complex) four-dimensional space S_4. The infinite group G of all such correspondences is obviously *not* the conformal group of S_4. Poincaré in his fundamental paper in Palermo Rendiconti (1907) has called G the group of *regular* transformations. In an abstract presented to the Society, 1908, Kasner found it more appropriate to term it the *pseudo-conformal group G*.

In a preceding paper, Kasner has given a purely geometric characterization. His main result is that *the pseudo-conformal group G is characterized by the fact that it leaves invariant the pseudo-angle between any curve and any hypersurface at their point of intersection.*[1]

In the present work, we shall find *all* the differential invariants of *first* order between the curves, surfaces, and hypersurfaces at a given point under the pseudo-conformal group. We shall take every combination of any two elements—six possible cases.[2] The number of independent invariants may be 0, 1, or 2.

A general pair of curve elements possesses no invariants. However, in the special case of an isoclinal pair, there is a unique invariant (the angle between them). A similar result is true for two hypersurface elements.

A hypersurface element and a curve element possess only one invariant—the *pseudo-angle*.[1]

To any general surface element S, there is associated a quadric regulus R of curve elements. There are no invariants between a general surface element S and a curve element e which is not on the regulus R of S. On the other hand, if e is in R, then there is a unique

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[1] Kasner, *Conformality in connection with functions of two complex variables*, Transactions of this Society, vol. 48 (1940), pp. 50–62. See also the following paper: Kasner, *Biharmonic functions and certain generalizations*, American Journal of Mathematics, vol. 58 (1936), pp. 377–390.

[2] We shall denote by e a curve element, that is, a lineal element; by S a general surface element; and by π a hypersurface element. The six possible cases are (e, e), (e, π), (π, π), (e, S), (π, S), (S, S).